Standards of Practice No. 7

MISSOURI COORDINATE SYSTEM OF 1983

A Manual for Land Surveyors

Revised 1996

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STATE COORDINATES FOR LAND SURVEYORS

The Manual of the Missouri Coordinate System of 1983
Standards of Practice No. 7

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INTRODUCTION

The Missouri State Coordinate System was developed in 1933 by Dr. O.S. Adams, mathematician in the Division of Geodesy, United States Coast and Geodetic Survey. This coordinate system was adopted as the official coordinate system for the State of Missouri by the Missouri Legislature in 1965. In 1984 the statute was revised to add the definition of the Missouri Coordinate System of 1983. The statute designates two legal systems, the older system based upon the Clarke Spheroid of 1866 and the North American Datum of 1927 and the newer system based upon the GRS 1980 and the North American Datum 1983. Either system was legally used until July 1990, but after that date only the new system may be used. The Missouri Department of Natural Resources' Division of Geology and Land Survey, Land Survey program supports only the Missouri Coordinate System of 1983; therefore, this manual is limited to that system.

The proper use of the State Coordinate System depends upon three factors:

1) The surveyor's knowledge of the system;
2) The end user's knowledge of the system; and
3) The existence of accurate horizontal control on which to base coordinate determinations.

The purpose of this manual is to meet the first two factors. The third factor is the responsibility of the Land Survey Program of the Department of Natural Resources.

CHAPTER 1
THE BASIC RECTANGULAR PLANE COORDINATE CONCEPT

The idea of combining the analytical methods of algebra and trigonometry with the concepts of geometry by means of a coordinate system was due largely to Descartes, a French mathematician. In 1637 he published the first systematic work on this subject under the title *La Geometrie.* The subject is sometimes called Cartesian geometry after the Latinized name of Descartes, Carteslus. The rectangular coordinates used in your courses on algebra and trigonometry are called Cartesian coordinates. Webster's New World Dictionary defines the Cartesian coordinates as 1) a pair of numbers that locate a point by its distance from two intersecting, often perpendicular, lines in the same plane; each distance is measured parallel to the other line; and 2) a triad of numbers which locate a point by its distance from three fixed planes that intersect one another at right angles.

The basic concept of the Cartesian Coordinate System is straight forward (fig.1). There are two mutually perpendicular lines, XX' and YY' called the X and Y axes. The axes intersect at point O called the origin. We may regard the line OX as east-west and the line OOY as north-south. We can choose a unit of measure and measure or mark those units and the axis with zero at the origin.

On the horizontal line we put positive numbers to the right and negative numbers to the left. On the vertical line we put positive numbers up and negative numbers down. In this way we designate that a positive direction on a horizontal line is to the right and on a vertical line as upward. Let's consider any point P in the plane of the axes. Its distance from the Y axis is called its X or east coordinate. This distance is positive if P is to the right of the Y axis, and negative if to the left. The distance of point P from the X axis is called its Y or north coordinate. This distance is positive if P is above the X axis and negative if below. These two coordinates together are called the rectangular coordinate or, simply, the coordinates of P.

In surveying and mapping the same type of Cartesian Coordinate System is used and the horizontal coordinates are referred to as the Rectangular Plane Coordinate System or Grid. The axes are north (up) and east (right); therefore, a point has a north and east coordinate. It can be readily seen that if we have the coordinate axes defined, and if we know the coordinates of a point, that point is defined in that plane. A properly drawn survey plat is the same as a graph. Each of the points which define the lines and corners of a project, therefore, can be defined by coordinates as long as reference axes are defined.

Figure 2 is an example of a lot survey using coordinates. The X and Y coordinate axes have been established in the plane of the paper, and the various coordinate values are shown for each one of the corner points. The use of a basic coordinate system such as this is quite common today. Most surveyors, draftsmen, and others use this basic concept for plotting maps and drawings. Computerized geographic information systems also use coordinates as a frame of reference. In order to plot data from computer files, the coordinate values of the points to be plotted must be known.

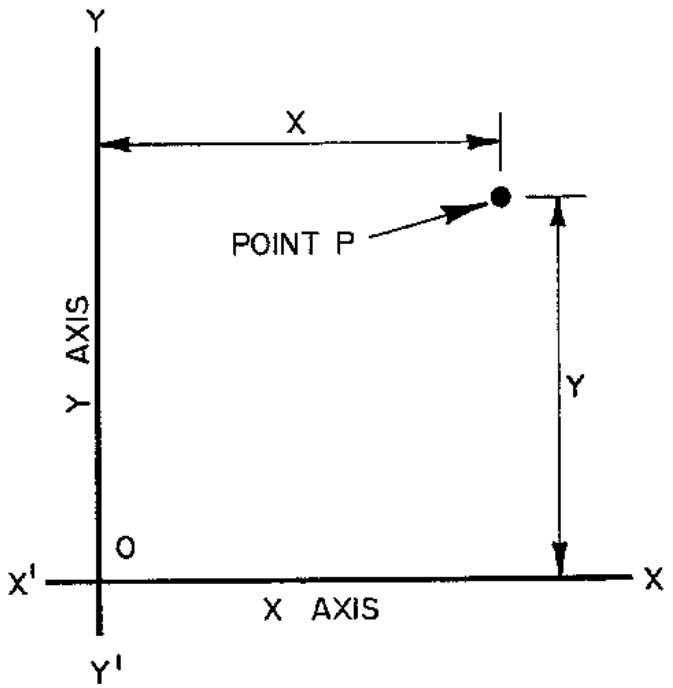


Figure 1. Example of Cartesian Coordinates

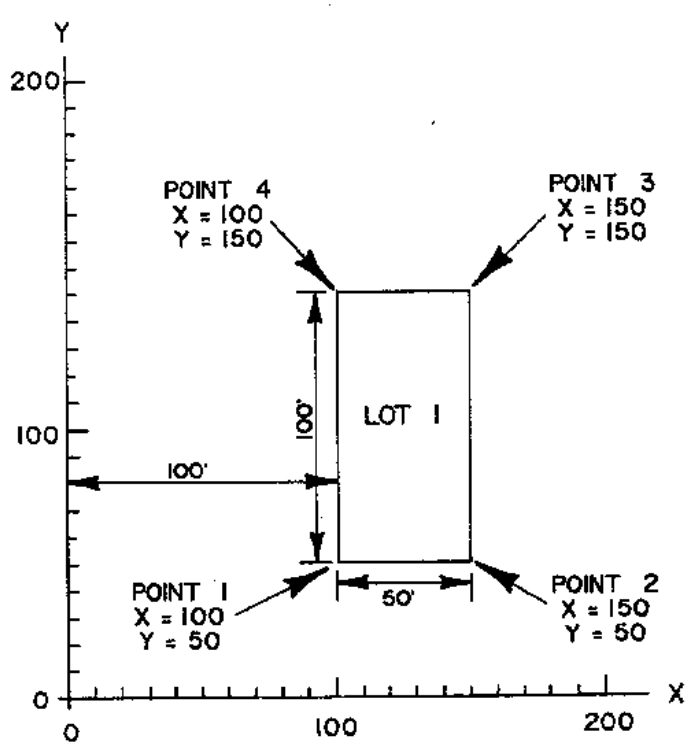


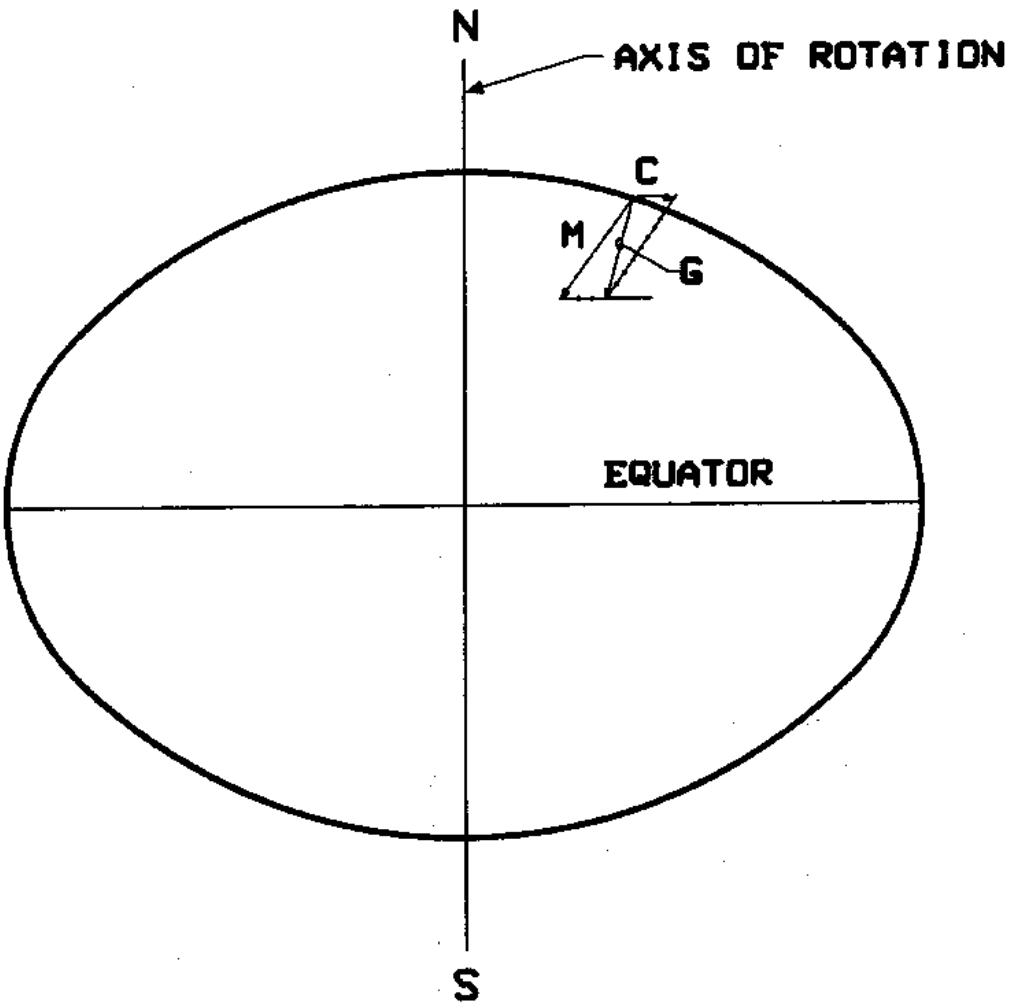
Figure 2. Example of Lot Survey

The coordinate system is not an abstract concept to most surveyors today. Coordinates are being used in all surveying and mapping. An individual plane coordinate system may be defined by one point having known coordinates and directions of the coordinate axes. The system may also be defined by two points having known coordinates. It is not necessary to know the physical location of the coordinate axes. Many computing and plotting programs used by surveyors, engineers, and mappers, require the adoption of a coordinate system. Usually one point, the starting point, is assigned positive X and Y coordinates and a direction from the starting point to the next point in the system is assigned. In this way the coordinate system is defined.

It is important to note that these assumed or local coordinate systems are adequate for the specific area or application but are limited in their use. In order to have a uniform system that can be expanded to cover adjoining surveys, cities, or counties and even adjoining states, a common system must be used.

The State Coordinate System provides a common datum for referencing the horizontal position of all surveys in the same way that mean sea level has provided a common datum for vertical position. It avoids the problem of having several surveys in an area, each based on its own assumed coordinate system, none of which are related to each other or to any other survey. The legally defined State Coordinate System is based upon a defined geodetic reference system. This reference system must possess the following features:

1) A collection of permanently marked and maintained points;
2) Coverage of an extensive area;
3) A spatial relationship of known accuracy;
4) Relationships expressed in a common mathematical language or in a language translatable into other languages; and
5) Universal availability of geodetic information.



M = GRAVITATIONAL FORCE
C = CENTRIFUGAL FORCE
G = GRAVITY (THE RESULTANT FORCE)

Figure 3. Ellipsoid of Revolution

CHAPTER 2
THE CURVED EARTH

The land surveyor is not normally concerned with the fact that his work is taking place on a curved surface. The modern surveyor has increased his technical ability to measure distances, directions, and astronomical azimuths to the point that the effect of curvature is a measurable quantity. This increased ability means that the professional surveyors must have an understanding of the curved surface on which he measures.

The earth is similar to a semi-fluid mass which is spinning on its axis. Every particle in this mass is acted upon by two forces. They are a gravitational force and centrifugal force. The resultant of these two forces is the force of gravity. Centrifugal force is greatest at the equator and therefore, the semi-fluid takes the shape of an ellipsoid of revolution. The shape is not a pure ellipsoid because of the varying density and stiffness of the crust (fig. 3).

When a surveyor uses a spirit level to determine elevation he is using a reference surface that is not a plane but a curved surface perpendicular to the force of gravity. The direction of the force of gravity does not point to the center mass of the earth but varies according to the values of gravitation and centrifugal forces. This surface is called the geoid and is an irregular surface (fig. 4).

It is impossible to compute or define a system of coordinates on an irregular surface like the geoid; therefore, a mathematical surface must be chosen that closely represents the shape of the geoid. The selection of the "best fit" ellipsoid has been a major concern of geodesists since the early 18th century.

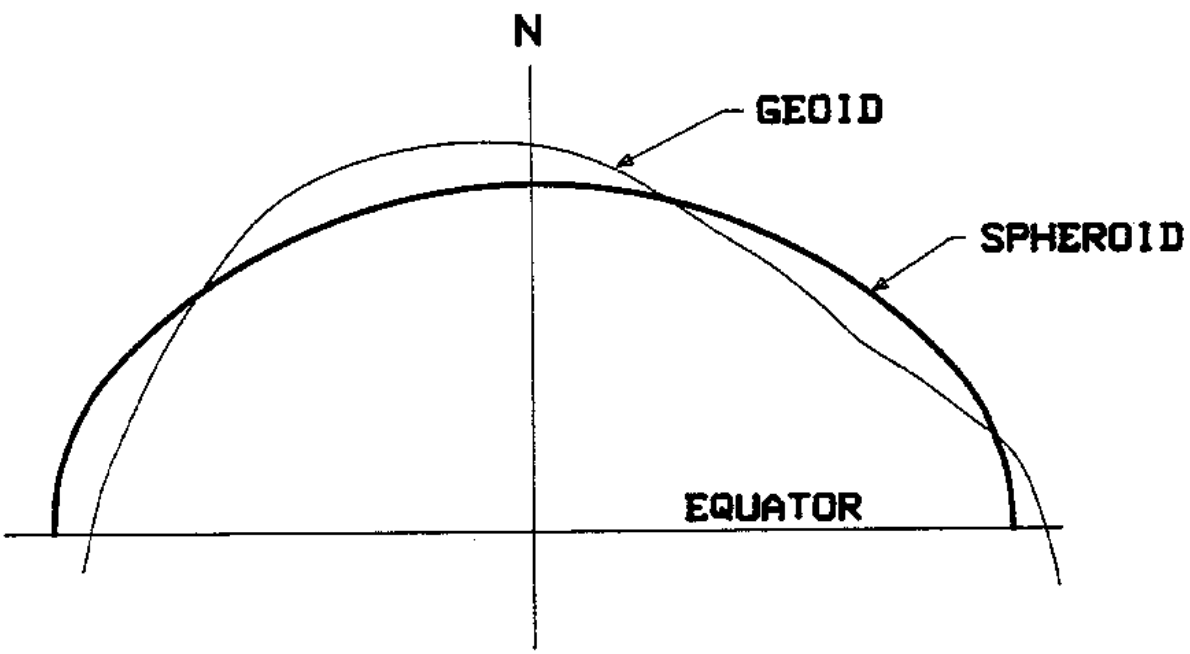


Figure 4. Geoid - Spheroid

Satellite data has provided geodesists with new measurements to define the best earth-fitting ellipsoid and for relating existing coordinate systems to the earth's center of mass. The ellipsoid used in the North American Datum 1983 (NAD83) was adopted by IUGG in 1980 and is called Geodetic Reference System 80 (GRS80) (fig. 5). The dimensions of GRS80 are:

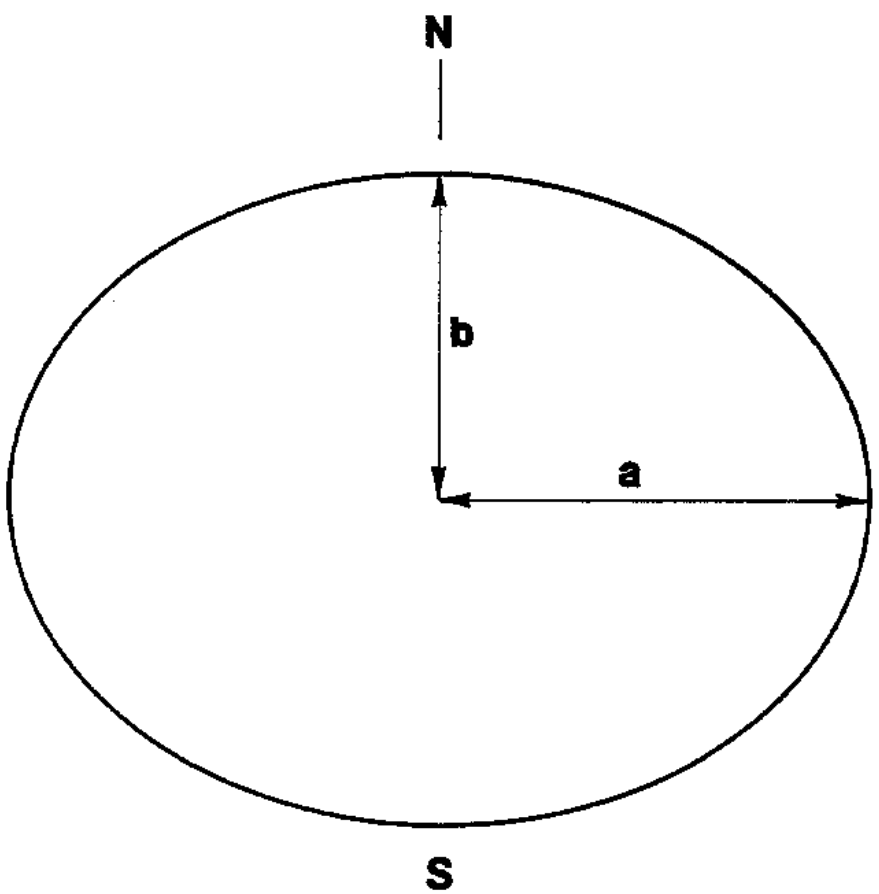
Equatorial semi-axis = a = 6,378,137.0 m (exact by definition)

Polar semi-axis = b = 6,356,752.3 m (computed)

The term eccentricity of the ellipse is defined to 10 significant digits as:

For GRS80 $e^2 = 0.0066943800$

The geocentric gravitational constant of earth, dynamic form factor of earth, and angular velocity are also defined. The semi-minor axis (b), flattening (f), and all other constants are computed from the defined constants, and should be given to a stated number of significant digits.



a = EQUATORIAL SEMI AXIS
b = POLAR SEMI AXIS

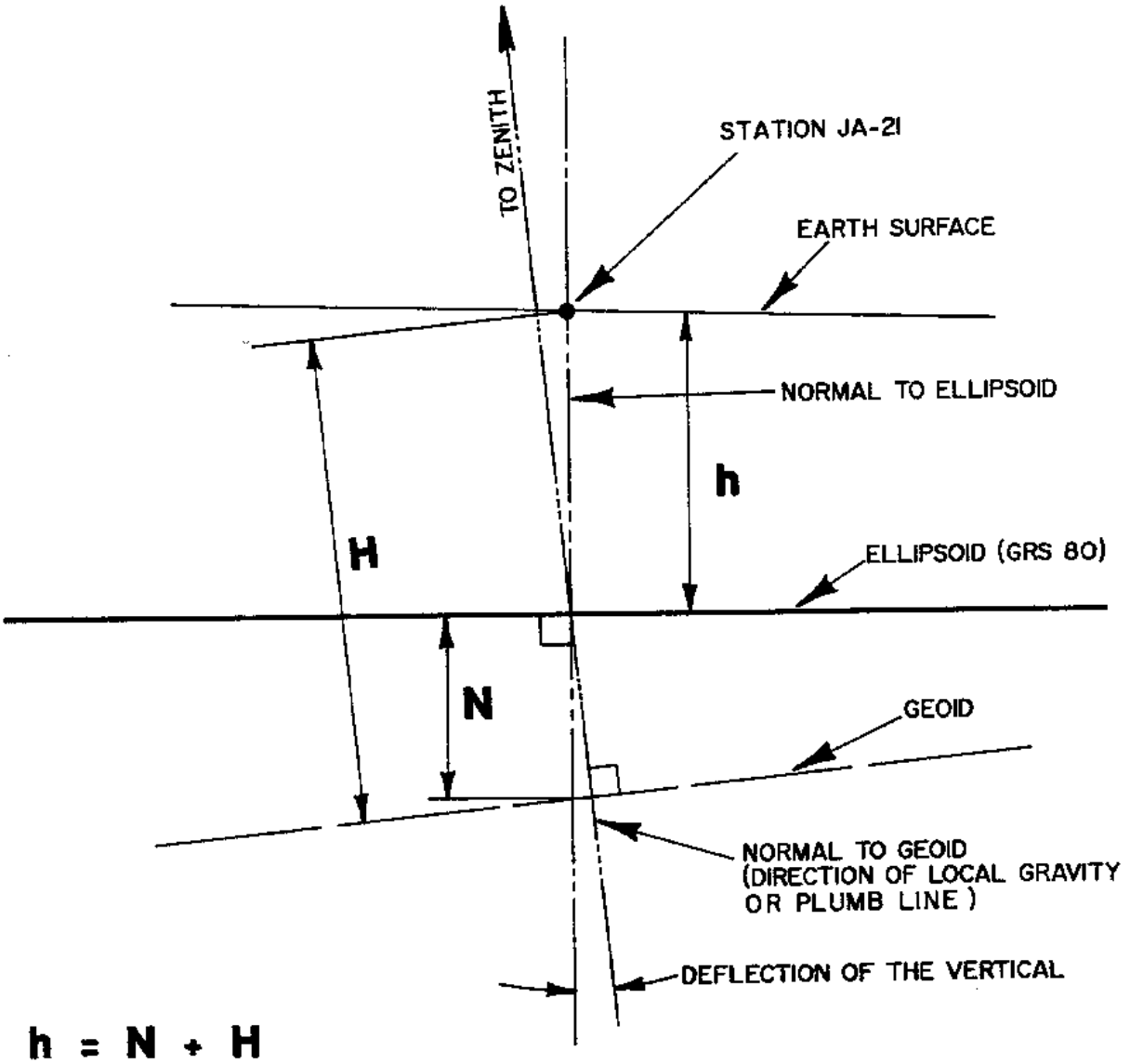
Figure 5. Dimensions of Ellipsoid

The relative position of the ellipsoid, the geoid, and the surface of the earth is important. Look at a small segment of the earth in Missouri at Station JA-21 (fig. 6). Curvature is not apparent at this single station. Three planes are shown - ellipsoid (GRS80), ground surface, and the geoid (NAVD88). The normal to the geoid is the direction of local gravity or plumb line. The normal to the ellipsoid is the vertical of the computational surface. The angle between these two verticals is called the deflection of the vertical and varies from 1 to 3 seconds in Missouri.

The orthometric height (H) which is very nearly the elevation of the ground surface is the distance measured from the geoid to the ground surface. Ellipsoidal height is the distance from the ellipsoid to the ground surface. Geoid separation is the distance from the ellipsoid to the geoid. Note that measurements up from the ellipsoid are positive and down negative.

In the state of Missouri the geoid separations are all negative. The geoid averages approximately 30.5 m below the ellipsoid.

It is important to note that elevations on the earth are reported relative to the geoid, not the ellipsoid. Latitude and longitude, on the other hand, are determined with respect to the ellipsoid. The height obtained from GPS observation is the ellipsoidal height.



$$h = N + H$$

H = ORTHOMETRIC HEIGHT OR ELEVATION
(DISTANCE FROM GEOID TO EARTH SURFACE)

h = ELLIPSOIDAL HEIGHT OR GEODETIC HEIGHT
(DISTANCE FROM ELLIPSOID TO EARTH SURFACE)

N = GEOID SEPARATION OR GEOID HEIGHT
(DISTANCE FROM ELLIPSOID TO GEOID)

Figure 6. Surfaces at Station JA-21

CHAPTER 3
REPRESENTATION OF THE CURVED SURFACE ON A FLAT PLANE (THE TRANSVERSE MERCATOR PROJECTION)

The earth on which all survey work is performed is essentially a sphere. Map makers for at least 500 years have attempted to prepare a flat map of a curved surface with as little distortion as possible.

A map projection is a systematic representation of all or part of the surface of a round body, especially the earth, on a plane. A map projection also serves as the basis for the development of a coordinate system that represents a curved surface on a plane. The transverse Mercator projection has been adopted for the Missouri Coordinate System of 1983. Therefore, in order to understand the coordinates we need to understand the characteristics of this projection.

The transverse Mercator projection was invented by the mathematician and cartographer Johann Heinrich Lambert in 1772. This projection uses the concepts of the Mercator projection developed by Gerardus Mercator in 1569.

In the transverse Mercator projection, a cylinder is wrapped around the sphere so that it touches a central meridian throughout its length. The other meridians and parallels of latitude are projected on the cylinder. The cylinder is cut along some parallel and rolled flat showing this projection. The central meridian can be made true scale no matter how far north the projection extends north and south.

In the general form of the transverse Mercator projection, one selected "central meridian" is rectified into a straight-line segment (N-S in fig. 7). The equator is represented by a straight line perpendicular to the central meridian. The other meridians and parallels of latitudes are curved lines of a complex nature. The central meridian becomes the Y axis and the equator becomes the X axis.

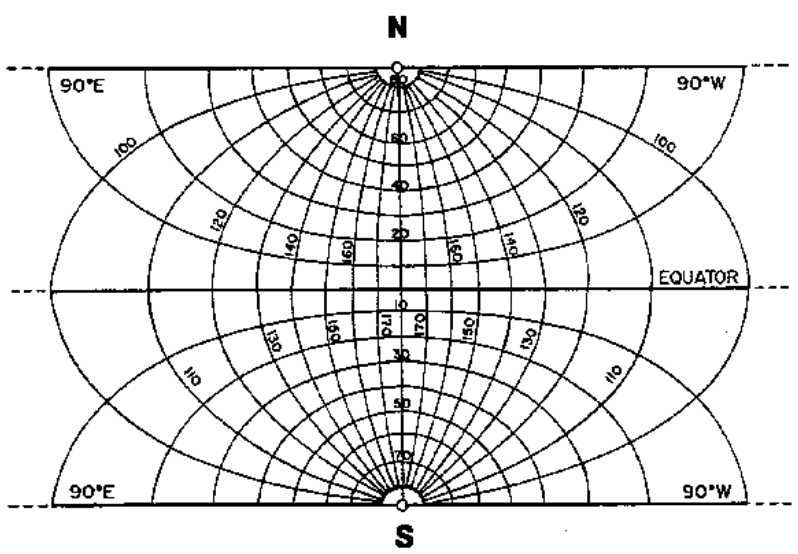
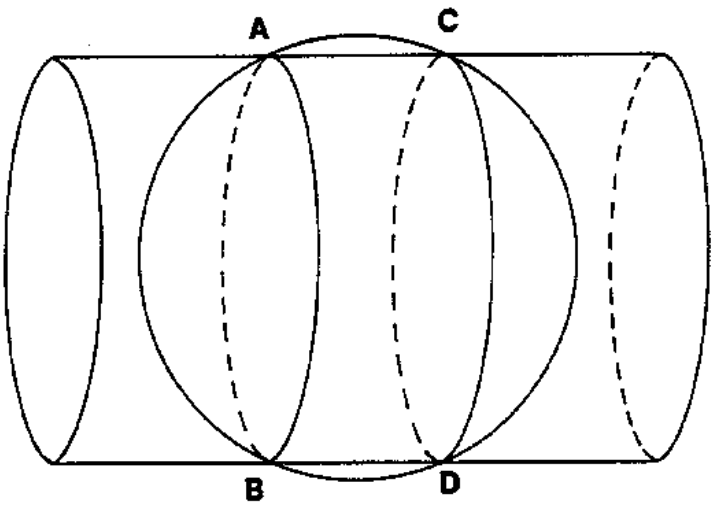
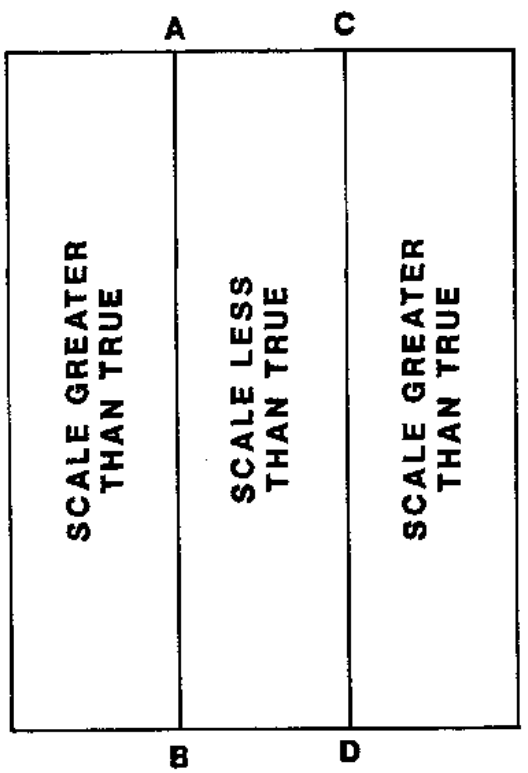


Figure 7. The Transverse Mercator Projection



DEVELOPABLE CYLINDER

Figure 8. Developable Cylinder



DEVELOPED CYLINDER

Figure 9. Developed Cylinder

The basic projection employed in the Missouri Coordinate System of 1983 is the transverse Mercator projection. It uses an imaginary cylinder as its developable surface, which is illustrated in figures 8 and 9. The cylinder is placed secant to the spheroid in the State Coordinate System. That is, it intersects the spheroid in two places - as along lines AB and CD in figure 8. Figure 9 illustrates the front half of the plane surface developed from the cylinder.

A section showing the relationship of the projected surface and the ellipsoid is taken perpendicular to the central meridian at the approximate center of the north-south range of the system (fig. 10).

In computing projections, points are projected mathematically from the ellipsoid along lines normal to the ellipsoid to the surface of the imaginary cylinder. Figure 10 illustrates graphically the projection of points, and also illustrates the relative length of a line on the ellipsoid and the length of that line when projected to the surface of the cylinder. Note for example, that the distance G"H" on the projection surface is greater than G'H' on the ellipsoid. It can be seen that the map projection scale is greater than the true spheroid scale where the cylinder is outside the ellipsoid. Conversely the distance A" B" on the projection is less than A'B' on the ellipsoid. The map scale is less than true spheroid scale where the projection surface is inside the ellipsoid. Points F and E occur at the intersection of the projection surface and the ellipsoid surface, and it may therefore be stated that map scale equals true ellipsoid scale along these lines of intersection.

As illustrated, it is impossible to project points from the ellipsoid to any developable surface without introducing distortions in the length of lines or in the shapes of areas. These distortions are held to a minimum, however, by placing the cylinder secant and by limiting the size of the zone or the extent of coverage of the earth's surface for any one projection. If the width of the zones are held to 158 miles or less, distortions are held to one part in 10,000 or less.

It will be noted that the state is divided into three zones. The maximum distortion at the central meridian has been held to one part in 15,000 too small in both the Eastern and Central Zones, and one part in 17,000 too small in the Western Zone.

The central meridian, in the exact center of the zone, defines the direction of the rectangular coordinate system. The grid direction is equal to geodetic north at the central meridian. The central meridian is also assigned a large X value or false easting so that all X values in the zone will be positive. The Y coordinate value is kept positive by placing the X axis, or origin, south of the state boundary.

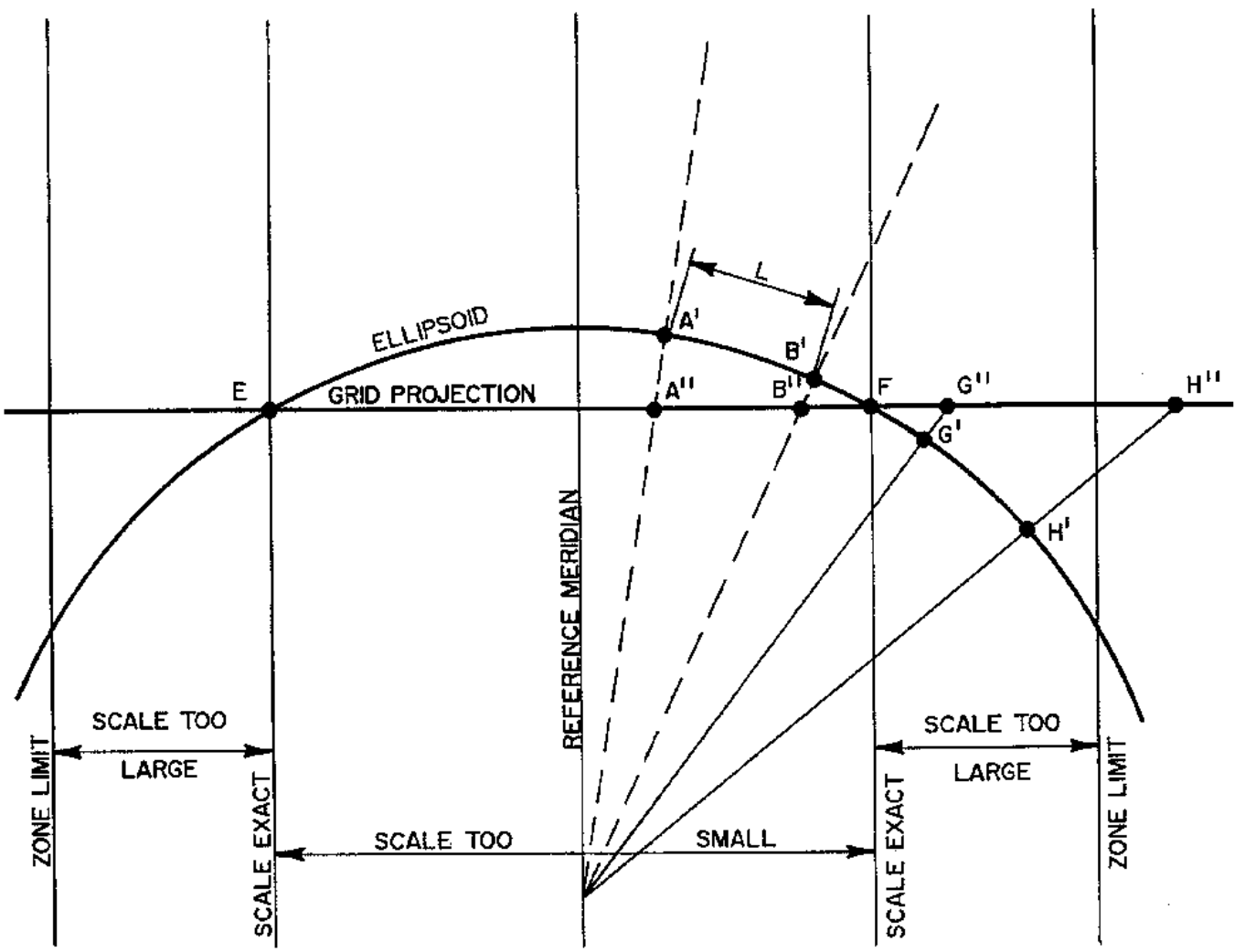


Figure 10. Section of Ellipsoid

CHAPTER 4
THE MISSOURI COORDINATE SYSTEM OF 1983

The Missouri Coordinate System of 1983 is defined by Missouri State Statute. This statute was first passed in 1965 and revised in 1984. The statute is as follows:

MISSOURI STATE COORDINATE SYSTEM

60.401. Missouri state coordinate system established. *- The systems of plane coordinates which have been established by the National Ocean Survey/National Geodetic Survey, or its successors, for defining and stating the geographic positions or locations of points on the surface of the earth within the state of Missouri are hereafter to be known and designated as the "Missouri Coordinate System of 1927" and the "Missouri Coordinate System of 1983."*

60.410 State divided into three zones-descriptions. *- 1. For the purpose of the use of this system, Missouri is divided into three separate zones, to be officially known as "The East Zone," "The Central Zone," and "The West Zone." (See fig. 11).*

2) The area now included in the following counties, shall constitute the east zone: Bollinger, Butler, Cape Girardeau, Carter, Clark, Crawford, Dent, Dunklin, Franklin, Gasconade, Iron, Jefferson, Lewis, Lincoln, Madison, Marion, Mississippi, Montgomery, New Madrid, Oregon, Perniscot, Perry, Pike, Ralls, Reynolds, Ripley, St. Charles, Ste. Genevieve, St. Francois, St. Louis, St. Louis (city), Scott, Shannon, Stoddard, Warren, Washington, and Wayne.

3) The area now included in the following counties shall constitute the central zone: Adair, Audrain, Benton, Boone, Callaway, Camden, Carroll, Chariton, Christian, Cole, Cooper, Dallas, Douglas, Greene, Grundy, Hickory, Howard, Howell, Knox, Laclede, Linn, Livingston, Macon, Maries, Mercer, Miller, Moniteau, Monroe, Morgan, Osage, Ozark, Pettis, Phelps, Polk, Pulaski, Putnam, Randolph, Saline, Schuyler, Scotland, Shelby, Stone, Sullivan, Taney, Texas, Webster, and Wright.

4. The area now included in the following counties shall constitute the west zone: Andrew, Atchison, Barry, Barton, Bates, Buchanan, Caldwell, Cass, Cedar, Clay, Clinton, Dade, Daviess, DeKalb, Gentry, Harrison, Henry, Holt, Jackson, Jasper, Johnson, Lafayette, Lawrence, McDonald, Newton, Nodaway, Platte, Ray, St. Clair, Vernon, and Worth.

60.421. Zones, official names. *- 1. As established for use in the east zone, the Missouri coordinate system of 1927 or the Missouri coordinate system of 1983 shall be named; and, in any land description in which it is used, it shall be designated the "Missouri Coordinate System of 1927, East Zone" or "Missouri Coordinate System of 1983, East Zone."*

2. As established for use in the central zone, the Missouri coordinate system of 1927 or the Missouri coordinate system of 1983 shall be named; and, in any land description in which it is used, it shall be designated the "Missouri Coordinate System of 1927, Central Zone" or "Missouri Coordinate System of 1983, Central Zone."

3. As established for use in the west zone, the Missouri coordinate system of 1927 or the Missouri coordinate system of 1983 shall be named; and, in any land description in which it is used, it shall be designated the "Missouri Coordinate System of 1927, West Zone" or "Missouri Coordinate System of 1983, West Zone."

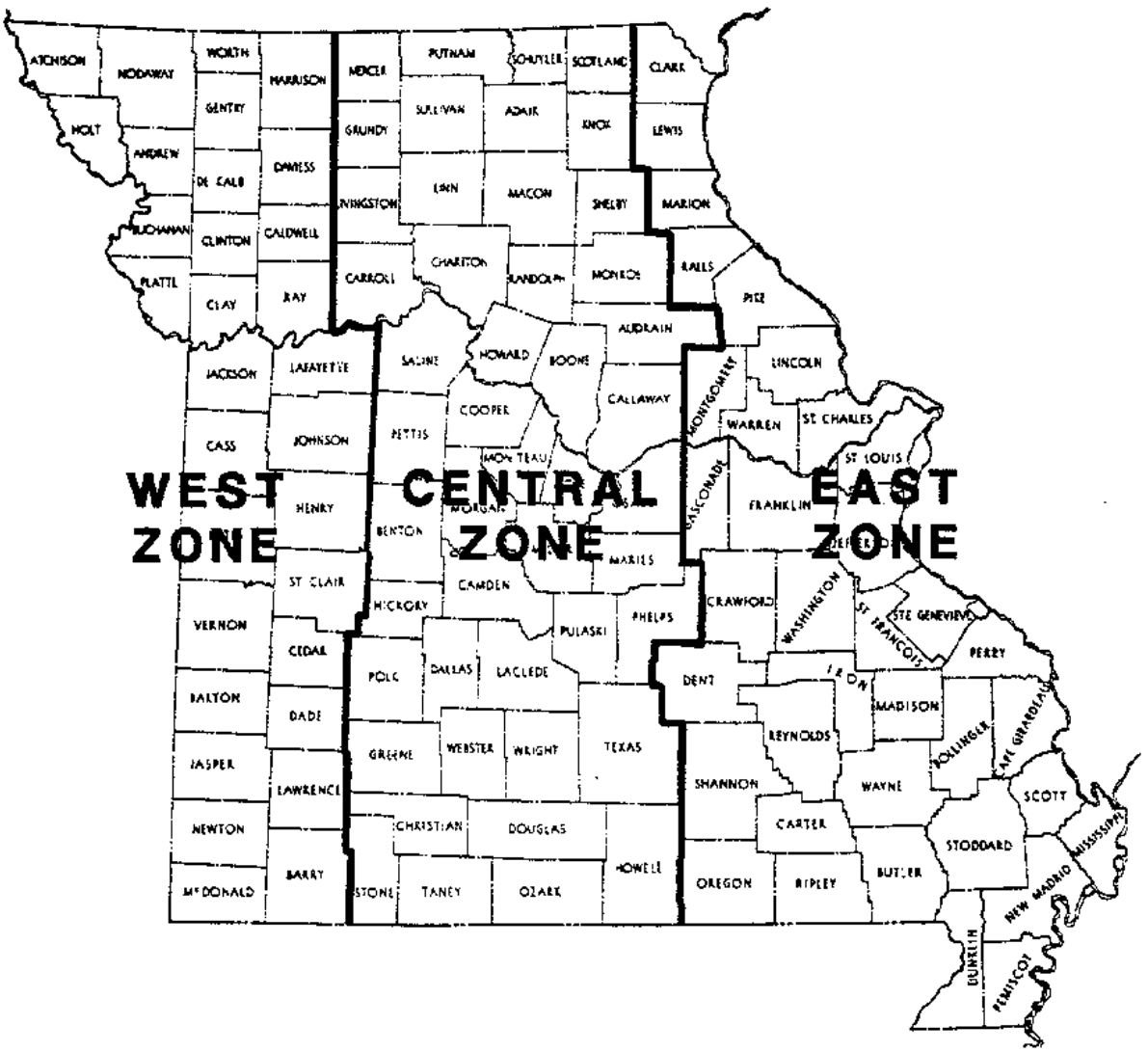


Figure 11. Missouri Coordinate System Zones

60.431. Location use of plane coordinate, to establish. *- The plane coordinate values for a point on the earth's surface, used to express the geographic position or locations of such point in the appropriate zone of this system, shall consist of two distances expressed in U.S. Survey Feet and decimals of a foot when using the Missouri coordinate system of 1927 and expressed in meters and decimals of a meter when using the Missouri coordinate system of 1983. One of these distances, to be known as the "x-coordinate," shall give the position in an east-and-west direction; the other, to be known as the "y-coordinate," shall give the position in a north-and-south direction. These coordinates shall be made to depend upon and conform to plane rectangular coordinate values for the monumented points of the North American Horizontal Geodetic Control Network, as published by the National Ocean Survey/ National Geodetic Survey, or its successors, and whose plane coordinates have been computed on the systems defined in sections 60.401 to 60.481. Any such station may be used for establishing a survey connection to either Missouri coordinate system.*

60.441. Descriptions involving more than one zone. *- When any tract of land to be defined by a single description extends from one into another of the coordinate zones set out in section 60.410, the positions of all points on its boundaries may be referred to as either of the zones and the zone which is used shall be specifically named in the description.*

60.451. Missouri coordinate system zones precisely defined. *- 1. For the purpose of more precisely defining the Missouri coordinate system of 1927, the following definition by the United States Coast and Geodetic Survey is adopted:*

(1) The Missouri coordinate system of 1927, east zone, is a transverse Mercator projection of the Clarke spheroid of 1866, having a central meridian 90 degrees-30 minutes west of Greenwich, on which meridian the scale is set at one part in fifteen thousand too small. The origin of coordinates is at the intersection of the meridian 90 degrees-30 minutes west of Greenwich and the parallel 35 degrees-50 minutes north latitude. This origin is given the coordinates: x = 500,000 feet and y = 0 feet;

(2) The Missouri coordinate system of 1927, central zone, is a transverse Mercator projection of the Clarke spheroid of 1866, having a central meridian 92 degrees-30 minutes west of Greenwich, on which meridian the scale is set at one part in fifteen thousand too small. The origin of coordinates is at the intersection of the meridian 92 degrees-30 minutes west of Greenwich and the parallel of 35 degrees-50 minutes north latitude. This origin is given the coordinates: x = 500,000 feet and y = 0 feet;

(3) The Missouri coordinate system of 1927, west zone, is a transverse Mercator projection of the Clarke spheroid of 1866, having a central meridian 94 degrees-30 minutes west of Greenwich, on which meridian the scale is set at one part in seventeen thousand too small. The origin of coordinates is at the intersection of the meridian 94 degrees-30 minutes west of Greenwich and the parallel 36 degrees-10 minutes north latitude. This origin is given the coordinates: x = 500,000 feet and y = 0 feet.

2. For purposes of more precisely defining the Missouri coordinate system of 1983, the following definition by the National Ocean Survey/National Geodetic Survey is adopted:

(1) The Missouri coordinate system 1983, east zone, is a transverse Mercator projection of the North American Datum of 1983 having a central meridian 90 degrees-30 minutes west of Greenwich, on which meridian the scale is set at one part in fifteen thousand too small. The origin of coordinates is at the intersection of the meridian 90 degrees-30 minutes west of Greenwich and the parallel 35 degrees-50 minutes north latitude. This origin is given the coordinates: x = 250,000 meters and y = 0 meters; (see fig. 12).

(2) The Missouri coordinate system 1983, central zone, is a transverse Mercator projection of the North American Datum of 1983 having a central meridian 92 degrees-30 minutes west of Greenwich, on which meridian the scale is set at one part in fifteen thousand too small. The origin of coordinates is at the intersection of the meridian 92 degrees-30 minutes west of Greenwich and the parallel of 35 degrees-50 minutes north latitude. This origin is given the coordinates: x = 500,000 meters and y = 0 meters; (see fig. 13).

(3) The Missouri coordinate system 1983, west zone, is a transverse Mercator projection of the North American Datum of 1983 having a central meridian 94 degrees-30 minutes west of Greenwich, on which meridian the scale is set at one part in seventeen thousand too small. The origin of coordinates is at the intersection of the meridian 94 degrees-30 minutes west of Greenwich and the parallel 36 degrees-10 minutes north latitude. This origin is given the coordinates: x = 850,000 meters and y = 0 meters (see fig. 14).

3. The position of either Missouri coordinate system shall be as marked on the ground by horizontal control stations established in conformity with the standards adopted by the department of natural resources for first-order and second-order work, whose geodetic positions have been rigidly adjusted on the appropriate datum and whose coordinates have been computed on the system defined in this section. Any such station may be used for establishing a survey connection with the Missouri coordinate system.

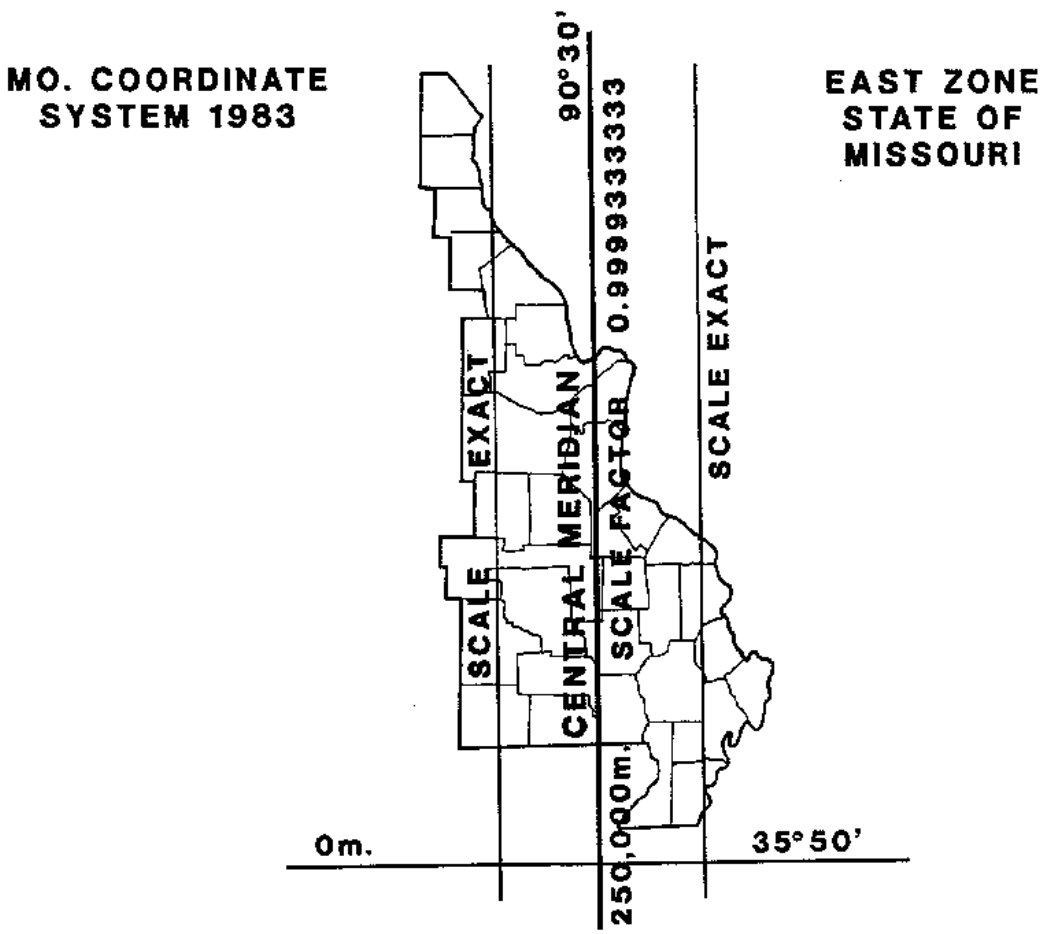


Figure 12. East Zone

60.461. Property descriptions not to be recorded unless containing a point within one kilometer of horizontal control station. *- No coordinates based on either Missouri coordinate system purporting to define the position of a land point on a land boundary shall be presented to be recorded in any public land records or deed records unless the point is within one kilometer of a horizontal control station established in conformity with the standards prescribed in section 60.451; except that, such one kilometer limitation may be modified by the department of natural resources to meet local conditions.*

60.471. Use of term limited. *- The use of the term "Missouri Coordinate System of 1927" or "Missouri Coordinate System of 1983" on any map, report of survey, or other document shall be limited to coordinates based on the Missouri coordinate system as defined in section 60.401 to 60.491.*

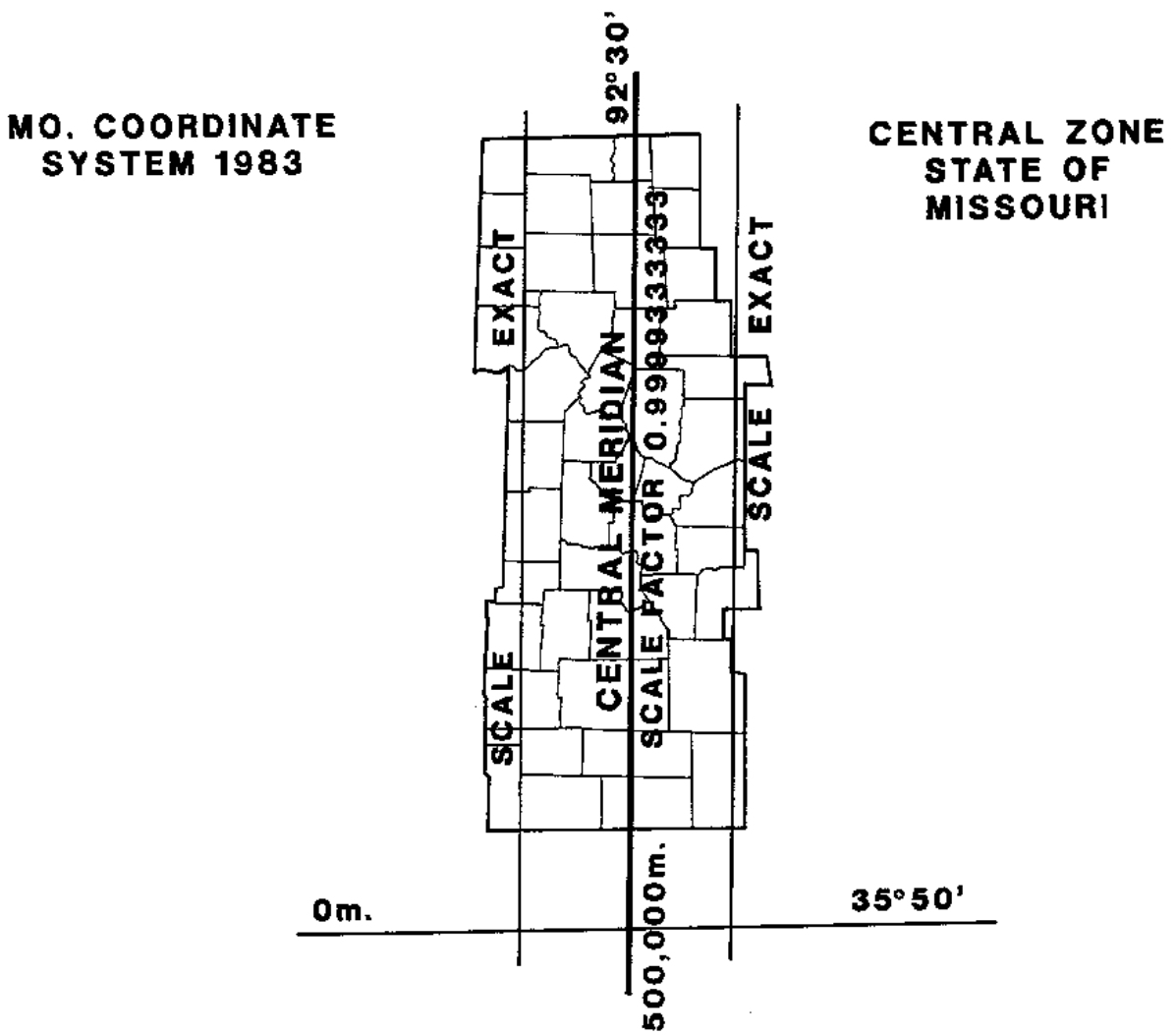


Figure 13. Central Zone

60.480. Property description based on United States public land survey recognized. *- Descriptions of tracts of land by reference to subdivisions, lines, or corners of the United States public land survey, or other original pertinent surveys, are hereby recognized as the basic and prevailing method for describing such tracts. Whenever coordinates of the Missouri coordinate system are used in such descriptions they shall be construed as being supplementary to descriptions of such subdivisions, lines, or corners contained in official plats and field notes of record; and, in the event of any conflict, the descriptions by reference to the subdivisions, lines, or corners of the United States public land surveys, or other original pertinent surveys shall prevail over the description by coordinates.*

60.491. Missouri coordinate system of 1983 to be sole system after July 1990. *- The Missouri coordinate system of 1927 shall not be used after July, 1990; and the Missouri coordinate system of 1983 shall be the sole system after this date.*

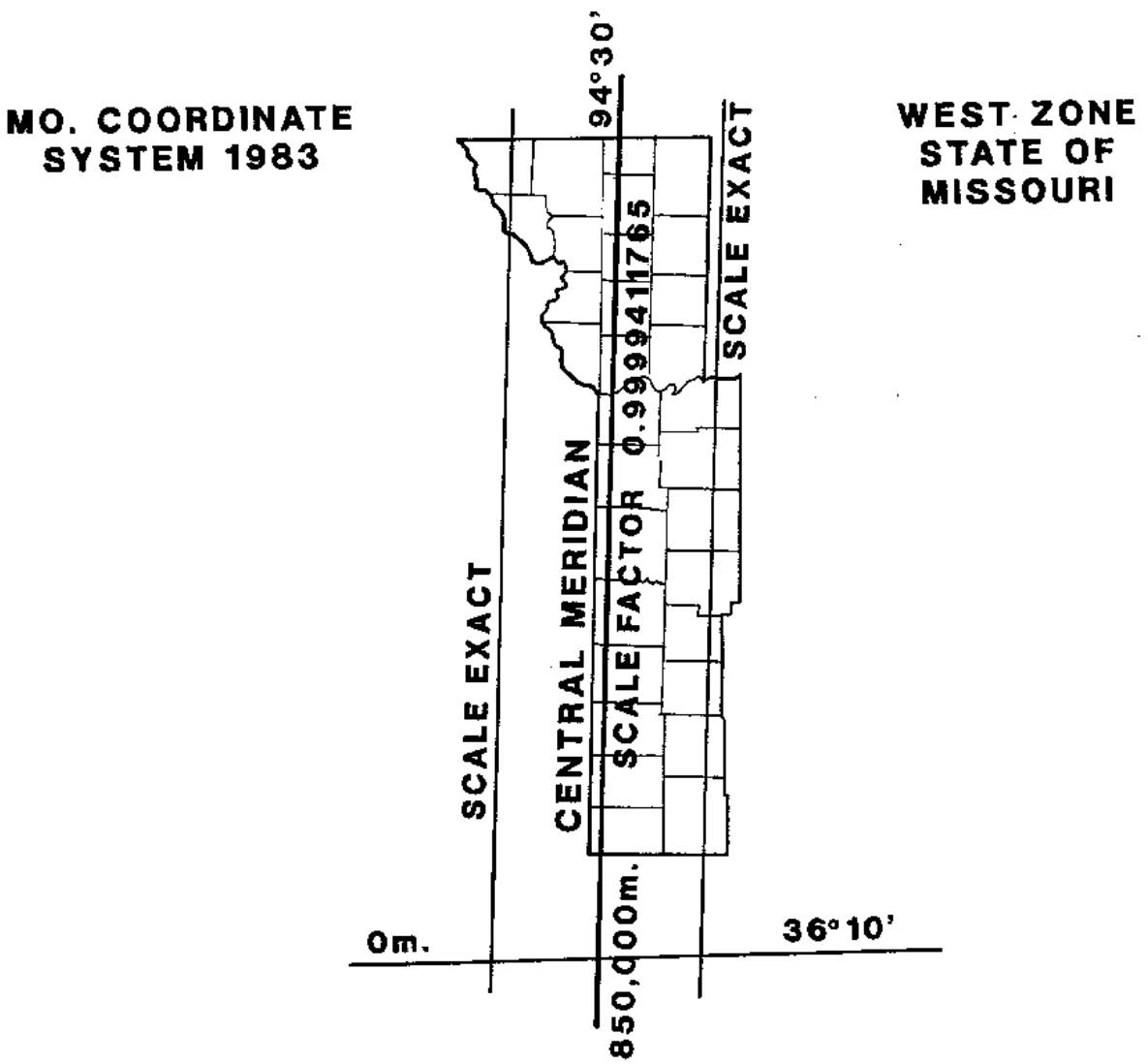


Figure 14. West Zone

CHAPTER 5

GRID DISTANCES, GRID BEARINGS, AND THEIR COMPUTATION MEASUREMENT OF DISTANCES

ELEVATION FACTOR

The surveyor measures all his distances on the earth's surface; therefore, the first step in using state plane coordinates is the reduction of these measurements to distances on the reference ellipsoid (fig. 15).

The general concept is to multiply the ground distance by a factor (table 1) which is dependent upon the mean elevation of the line being measured.

$$S = Sm \left(\frac{R}{R+H}\right)\left(1 - \frac{N}{R}\right)$$

S = Sm times elevation factor

$$\textit{Elevation factor} = \left(\frac{R}{R+H}\right)\left(1 - \frac{N}{R}\right)$$

Where S = Ellipsoid distance

Sm = Measured ground distance

R = Ellipsoid radius in feet

N = Geoid separation in feet

H = Mean elevation of measurements in feet

For Missouri Coordinate System 1983: *(H in feet)*

$$\textit{Elevation factor} = \left(\frac{20{,}909{,}689}{20{,}909{,}689 + H}\right)(1.00000479)$$

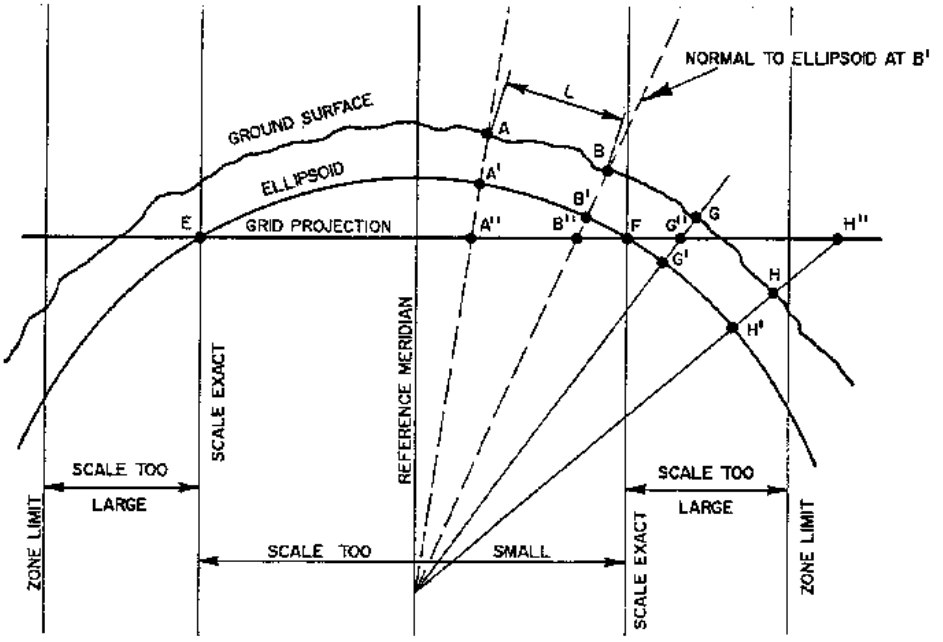


Figure 15. Section of Earth

TABLE 1 - ELEVATION FACTOR TABLE
(GEOID SEPARATION -30.5 METERS)

Elevation (feet)	Elevation Factor	Difference	Feet	Proportional Part of 0.00000024
50	1.0000024			
100	1.0000000	0.0000024	1	0.0000000
150	0.9999976	0.0000024	3	0.0000001
200	0.9999952	0.0000024	5	0.0000002
250	0.9999928	0.0000024	7	0.0000003
300	0.9999904	0.0000024	9	0.0000004
350	0.9999880	0.0000024	11	0.0000005
400	0.9999857	0.0000024	13	0.0000006
450	0.9999833	0.0000024	15	0.0000007
500	0.9999809	0.0000024	17	0.0000008
550	0.9999785	0.0000024	19	0.0000009
600	0.9999761	0.0000024	21	0.0000010
650	0.9999737	0.0000024	23	0.0000011
700	0.9999713	0.0000024	25	0.0000012
750	0.9999689	0.0000024	27	0.0000013
800	0.9999665	0.0000024	29	0.0000014
850	0.9999641	0.0000024	31	0.0000015
900	0.9999617	0.0000024	33	0.0000016
950	0.9999594	0.0000024	35	0.0000017
1000	0.9999570	0.0000024	37	0.0000018
1050	0.9999546	0.0000024	39	0.0000019
1100	0.9999522	0.0000024	41	0.0000020
1150	0.9999498	0.0000024	43	0.0000021
1200	0.9999474	0.0000024	45	0.0000022
1250	0.9999450	0.0000024	47	0.0000023
1300	0.9999426	0.0000024	49	0.0000024
1350	0.9999402	0.0000024	50	0.0000024
1400	0.9999378	0.0000024		
1450	0.9999354	0.0000024		
1500	0.9999331	0.0000024		
1550	0.9999307	0.0000024		
1600	0.9999283	0.0000024		
1650	0.9999259	0.0000024		
1700	0.9999235	0.0000024		
1750	0.9999211	0.0000024		
1800	0.9999187	0.0000024		
1850	0.9999163	0.0000024		
1900	0.9999139	0.0000024		
1950	0.9999115	0.0000024		
2000	0.9999091	0.0000024		
2050	0.9999068	0.0000024		

ERROR IN DISTANCE DUE TO ELEVATION FACTOR

Elevation Error	Error in Proportional Part	Error in ppm
50	1:416,667	2
100	1:208,333	5
200	104,167	10
300	52,083	19
400	26,042	38
500	13,021	77
600	6,510	154
700	3,255	307
800	1,628	614
900	814	1,229
1000	407	2,458
1100	203	4,915
1200	102	9,830
1300	51	19,661
1400	25	39,322
1500	13	78,643
1600	6	157,286

EXAMPLE CALCULATIONS FOR ELEVATION FACTOR (using table 1)

Station elevation in feet is	306	
From table for	300	0.9999904
Proportional part for	6	-0.0000002
Elevation factor		0.9999902
Station elevation in feet is	322	
From table for	300	0.9999904
Proportional part for	22	-0.0000010
Elevation factor		0.9999894
Station elevation in feet is	1005	
From table for	1000	0.9999570
Proportional part for		-0.0000003
Elevation factor		0.9999567
Station elevation in feet is	1056	
From table for	1050	0.9999546
Proportional part for	6	-0.0000002
Elevation factor		0.9999544

SCALE FACTOR

Distances on the reference ellipsoid must be projected to the plane surface of the coordinate system. As with the change of distances from the earth's surface to the reference ellipsoid, distances are multiplied by a factor called the scale factor (see fig. 15).

In the traverse Mercator projection the scale factor varies with the distance from the central meridian. The scale factor is a constant in a north-south direction.

For land surveys the scale factor is computed by the formula:

$M = M_o (1 + X''^2)$,

Where M = Scale factor

M_o = *Scale factor of central meridian*

$X'' = \frac{X'}{Q}$

$X' = X - X_o$ *or east coordinate of point - east coordinate of central meridian*

Q = Constant for each zone and ellipsoid

Table 2 shows the computed scale factors for various distances from the central meridian for the east and central zones.

Table 3 shows the computed scale factors for various distances from the central meridian for the west zone.

TABLE 2 - SCALE FACTOR - EAST AND CENTRAL ZONES
MISSOURI COORDINATE SYSTEM OF 1983

(East Zone - Central Meridian Equals 250,000 Meters)
(Central Zone - Central Meridian Equals 500,000 Meters)

X' Meters	M	Difference	X' Meters	M	Difference
0	0.9999333		61,500	0.9999799	0.0000022
1,500	0.9999334	0.0000000	63,000	0.9999822	0.0000023
3,000	0.9999334	0.0000001	64,500	0.9999845	0.0000024
4,500	0.9999336	0.0000001	66,000	0.9999869	0.0000024
6,000	0.9999338	0.0000002	67,500	0.9999894	0.0000025
7,500	0.9999340	0.0000002	69,000	0.9999919	0.0000025
9,000	0.9999343	0.0000003	70,500	0.9999945	0.0000026
10,500	0.9999347	0.0000004	72,000	0.9999971	0.0000026
12,000	0.9999351	0.0000004	73,500	0.9999998	0.0000027
13,500	0.9999356	0.0000005	75,000	1.0000026	0.0000027
15,000	0.9999361	0.0000005	76,500	1.0000054	0.0000028
16,500	0.9999367	0.0000006	78,000	1.0000082	0.0000029
18,000	0.9999373	0.0000006	79,500	1.0000111	0.0000029
19,500	0.9999380	0.0000007	81,000	1.0000141	0.0000030
21,000	0.9999388	0.0000007	82,500	1.0000171	0.0000030
22,500	0.9999396	0.0000008	84,000	1.0000202	0.0000031
24,000	0.9999404	0.0000009	85,500	1.0000233	0.0000031
25,500	0.9999413	0.0000009	87,000	1.0000265	0.0000032
27,000	0.9999423	0.0000010	88,500	1.0000297	0.0000032
28,500	0.9999433	0.0000010	90,000	1.0000330	0.0000033
30,000	0.9999444	0.0000011	91,500	1.0000364	0.0000034
31,500	0.9999455	0.0000011	93,000	1.0000398	0.0000034
33,000	0.9999467	0.0000012	94,500	1.0000433	0.0000035
34,500	0.9999480	0.0000012	96,000	1.0000468	0.0000035
36,000	0.9999493	0.0000013	97,500	1.0000503	0.0000036
37,500	0.9999506	0.0000014	99,000	1.0000540	0.0000036
39,000	0.9999521	0.0000014	100,500	1.0000577	0.0000037
40,500	0.9999535	0.0000015	102,000	1.0000614	0.0000037
42,000	0.9999550	0.0000015	103,500	1.0000652	0.0000038
43,500	0.9999566	0.0000016	105,000	1.0000690	0.0000038
45,000	0.9999583	0.0000016	106,500	1.0000729	0.0000039
46,500	0.9999599	0.0000017	108,000	1.0000769	0.0000040
48,000	0.9999617	0.0000017	109,500	1.0000809	0.0000040
49,500	0.9999635	0.0000018	111,000	1.0000850	0.0000041
51,000	0.9999653	0.0000019	112,500	1.0000891	0.0000041
52,500	0.9999673	0.0000019	114,000	1.0000933	0.0000042
54,000	0.9999692	0.0000020	115,500	1.0000975	0.0000042
55,500	0.9999712	0.0000020	117,000	1.0001018	0.0000043
57,000	0.9999733	0.0000021	118,500	1.0001062	0.0000043
58,500	0.9999755	0.0000021	120,000	1.0001106	0.0000044
60,000	0.9999776	0.0000022			

TABLE 3 - SCALE FACTOR - WEST ZONES
MISSOURI COORDINATE SYSTEM OF 1983

(West Zone - Central Meridian Equals 850,000 Meters)

X' Meters	M	Difference	X' Meters	M	Difference
0	0.9999412		61,500	0.9999878	0.0000022
1,500	0.9999412	0.0000000	63,000	0.9999901	0.0000023
3,000	0.9999413	0.0000001	64,500	0.9999924	0.0000024
4,500	0.9999414	0.0000001	66,000	0.9999948	0.0000024
6,000	0.9999416	0.0000002	67,500	0.9999973	0.0000025
7,500	0.9999419	0.0000002	69,000	0.9999998	0.0000025
9,000	0.9999422	0.0000003	70,500	1.0000024	0.0000026
10,500	0.9999426	0.0000004	72,000	1.0000050	0.0000026
12,000	0.9999430	0.0000004	73,500	1.0000077	0.0000027
13,500	0.9999434	0.0000005	75,000	1.0000104	0.0000027
15,000	0.9999770	0.0000005	76,500	1.0000132	0.0000028
16,500	0.9999446	0.0000006	78,000	1.0000161	0.0000029
18,000	0.9999452	0.0000006	79,500	1.0000190	0.0000029
19,500	0.9999459	0.0000007	81,000	1.0000220	0.0000030
21,000	0.9999466	0.0000007	82,500	1.0000250	0.0000030
22,500	0.9999474	0.0000008	84,000	1.0000280	0.0000031
24,000	0.9999483	0.0000009	85,500	1.0000312	0.0000031
25,500	0.9999492	0.0000009	87,000	1.0000344	0.0000032
27,000	0.9999502	0.0000010	88,500	1.0000376	0.0000032
28,500	0.9999512	0.0000010	90,000	1.0000409	0.0000033
30,000	0.9999523	0.0000011	91,500	1.0000443	0.0000034
31,500	0.9999534	0.0000011	93,000	1.0000477	0.0000034
33,000	0.9999546	0.0000012	94,500	1.0000511	0.0000035
34,500	0.9999559	0.0000012	96,000	1.0000546	0.0000035
36,000	0.9999572	0.0000013	97,500	1.0000582	0.0000036
37,500	0.9999585	0.0000014	99,000	1.0000618	0.0000036
39,000	0.9999599	0.0000014	100,500	1.0000655	0.0000037
40,500	0.9999614	0.0000015	102,000	1.0000693	0.0000037
42,000	0.9999629	0.0000015	103,500	1.0000731	0.0000038
43,500	0.9999645	0.0000016	105,000	1.0000769	0.0000038
45,000	0.9999661	0.0000016	106,500	1.0000808	0.0000039
46,500	0.9999678	0.0000017	108,000	1.0000848	0.0000040
48,000	0.9999696	0.0000017	109,500	1.0000888	0.0000040
49,500	0.9999714	0.0000018	111,000	1.0000929	0.0000041
51,000	0.9999732	0.0000019	112,500	1.0000970	0.0000041
52,500	0.9999751	0.0000019	114,000	1.0001012	0.0000042
54,000	0.9999771	0.0000020	115,500	1.0001097	0.0000042
55,500	0.9999791	0.0000020	117,000	1.0001097	0.0000043
57,000	0.9999812	0.0000021	118,500	1.0001140	0.0000043
58,500	0.9999833	0.0000021	120,000	1.0001184	0.0000044
60,000	0.9999855	0.0000022			

EXAMPLE CALCULATIONS FOR SCALE FACTOR

Station coordinates - north 348,962.558 meters, east 829, 520.371 meters
West Zone

East coordinate of central meridian	850,000 meters
Subtract east coordinate of station	829,520
X' =	20,480

From table

				M
For X'	=	19,500		0.9999459
Actual	=	20,480		
Part		980		
Proportional part			980/1500	
Percentage			65%	
Difference =			0.0000007	
Difference times percentage				0.0000005
Scale Factor				**0.9999464**

Station coordinates - north 303,646.220 meters, east 860,950.555 meters
West Zone

East coordinate of central meridian	850,000 meters
Subtract east coordinate of station	860,951
X' =	10,951

From table

				M
For X'	=	10,500		0.9999426
Actual	=	10,951		
Part		451		
Proportional part			451/1500	
Percentage			30%	
Difference =			0.0000004	
Difference times percentage				0.0000001
Scale Factor				**0.9999427**

Station coordinates - north 152,967.424 meters, east 429,608.566 meters
Central Zone

East coordinate of central meridian	500,000 meters
Subtract east coordinate of station	429,609
X' =	70,391

From table

				M
For X'	=	69,000		0.9999919
Actual	=	70,391		
Part		1,391		
Proportional part			1391/1500	
Percentage			93%	
Difference =			0.0000025	
Difference times percentage				0.0000023
Scale Factor				**0.9999942**

GRID FACTOR

The surveyor can combine the scale factor and the elevation factor to form one factor. The product of EF and M is called the Grid Factor. This factor is most often given and is required to be shown on a plat using state coordinates.

USE OF THE GRID FACTOR

In order to reduce the length of a line to the grid distance the ground length is multiplied by the average grid factor for that line. In most cases that grid factor is the average of the grid factors at each end of the line.

If the lines are long and in the east-west direction, the mean grid factor may be computed by the following formula.

$$GF_L = \frac{GF_1 + 4GF + GF_2}{6}$$

Where $\overline{GF}$ is the grid factor at the midpoint of the line.

EXAMPLE CALCULATIONS FOR GRID FACTOR

For station		
Elevation factor	0.9999902	
Scale factor	0.9999464	
Grid factor (EF x SF)	=	0.9999366
For station		
Elevation factor	0.9999894	
Scale factor	0.9999427	
Grid factor (EF x SF)	=	0.9999321

MEASUREMENT OF DIRECTIONS

Surveyors are already familiar with the use of astronomic, magnetic, and assumed meridians as a basis of the bearing system of their survey. In the state plane coordinate system, the basis of direction is the coordinate grid lines.

The relationship between grid and geodetic north must be understood. In the transverse Mercator projection, grid north and geodetic north are the same along the central meridian. All grid lines in the plane system are parallel or perpendicular to the central meridian. The true meridians, however, converge, and therefore, the grid meridian and true meridian coincide only at the central meridian (see figs. 16 and 17).

The amount that the grid north differs from geodetic north is called the grid convergence. The value of this convergence varies with the distance from the central meridian and the latitude.

This convergence can be computed from the geographic coordinates (latitude and longitude) or from the grid coordinates.

<u>Using Latitude and Longitude</u>:

$C = \Delta \lambda \sin \phi,$

Where

C = *Convergence*

$\Delta \lambda = \lambda - \lambda_o$

λ = *Longitude of point*

λ_o = *Longitude of central meridan*

ϕ = *Latitude of points*

Using grid coordinates:

$C = M x' + E (x')^3,$

Where

C = *Convergence (in seconds)*

$x' = x - x_o$

x = *East coordinate of point*

x_o = *East coordinate of central meridian*

M & E = Factors dependent upon Y or the north coordinate

Tables 4 and 5 show computed values of M and E for the Missouri Coordinate System of 1983.

TABLE 4 - CONVERGENCE TABLES - CENTRAL AND EAST ZONES
MISSOURI COORDINATE SYSTEM OF 1983

(East Zone - Central Meridian Equals 250,000 Meters)
(Central Zone - Central Meridian Equals 500,000 Meters)

North Coordinate	M Sec per Meter	Difference	E Sec per Meter
0	0.233257		-2.9E-16
20,000	0.0234804	0.0001547	-2.9E-16
40,000	0.0236359	0.0001554	-3.0E-16
60,000	0.0237920	0.0001562	-3.0E-16
80,000	0.0239489	0.0001569	-3.0E-16
100,000	0.0241065	0.0001576	-3.1E-16
120,000	0.0242648	0.0001583	-3.1E-16
140,000	0.0244239	0.0001591	-3.1E-16
160,000	0.0245837	0.0001598	-3.2E-16
180,000	0.0247443	0.0001606	-3.2E-16
200,000	0.0249057	0.0001614	-3.2E-16
220,000	0.0250678	0.0001621	-3.2E-16
240,000	0.0252307	0.0001629	-3.3E-16
260,000	0.0253944	0.0001637	-3.3E-16
280,000	0.0255589	0.0001645	-3.4E-16
300,000	0.0257243	0.0001653	-3.4E-16
320,000	0.0258904	0.0001662	-3.5E-16
340,000	0.0260574	0.0001670	-3.5E-16
360,000	0.0262252	0.0001678	-3.5E-16
380,000	0.0263939	0.0001687	-3.6E-16
400,000	0.0265634	0.0001695	-3.6E-16
420,000	0.0267339	0.0001704	-3.7E-16
440,000	0.0269051	0.0001713	-3.7E-16
460,000	0.0270773	0.0001722	-3.8E-16
480,000	0.0272504	0.0001731	-3.8E-16
500,000	0.0274244	0.0001740	-3.8E-16
520,000	0.0275993	0.0001749	-3.9E-16
540,000	0.0277752	0.0001759	-3.9E-16
560,000	0.0279520	0.0001768	-4.0E-16
580,000	0.0281298	0.0001778	-4.0E-16
600,000	0.0283085	0.0001787	-4.1E-16
620,000	0.0284882	0.0001797	-4.1E-16

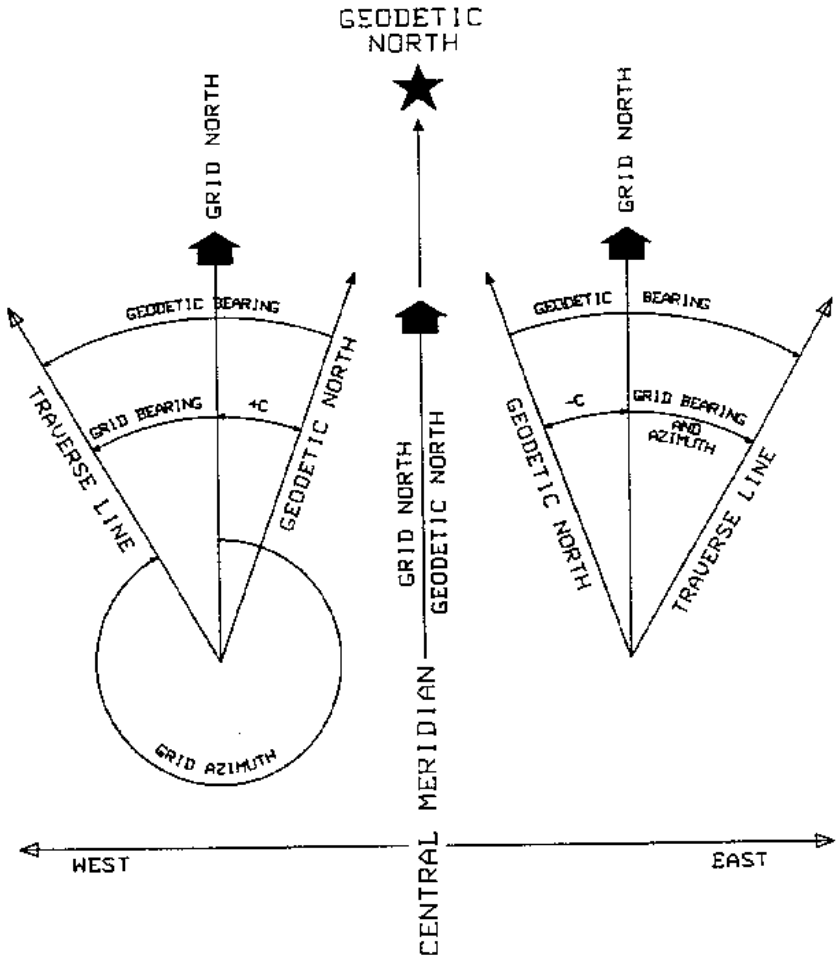


Figure 16. Grid Azimuth and Geodetic North

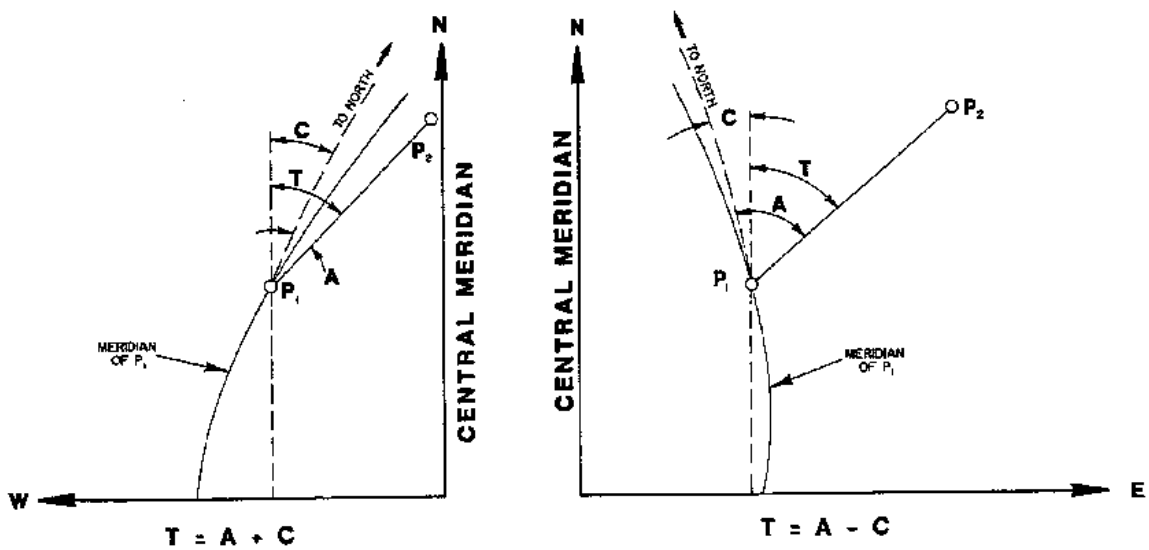


Figure 17. Grid Bearing and Geodetic North

TABLE 5 - CONVERGENCE TABLES - WEST ZONE
MISSOURI COORDINATE SYSTEM OF 1983

(West Zone - Central Meridian Equals 850,000 Meters)

North Coordinate	M Sec per Meter	Difference	E Sec per Meter
0	0.0236124		-3.0E-16
20,000	0.0237684	0.0001560	-3.0E-16
40,000	0.0239252	0.0001568	-3.0E-16
60,000	0.0240827	0.0001575	-3.0E-16
80,000	0.0242409	0.0001582	-3.1E-16
100,000	0.0243998	0.0001590	-3.1E-16
120,000	0.0245596	0.0001597	-3.2E-16
140,000	0.0247200	0.0001605	-3.2E-16
160,000	0.0248813	0.0001612	-3.2E-16
180,000	0.0250433	0.0001620	-3.3E-16
200,000	0.0252061	0.0001628	-3.3E-16
220,000	0.0253967	0.0001636	-3.3E-16
240,000	0.0255341	0.0001644	-3.4E-16
260,000	0.0256993	0.0001652	-3.4E-16
280,000	0.0258653	0.0001660	-3.5E-16
300,000	0.0260321	0.0001669	-3.5E-16
320,000	0.0261998	0.0001677	-3.5E-16
340,000	0.0263684	0.0001685	-3.6E-16
360,000	0.0265378	0.0001694	-3.6E-16
380,000	0.0267081	0.0001703	-3.7E-16
400,000	0.0268792	0.0001712	-3.7E-16
420,000	0.0270513	0.0001720	-3.8E-16
440,000	0.0272242	0.0001729	-3.8E-16
460,000	0.0273981	0.0001739	-3.8E-16
480,000	0.0275729	0.0001748	-3.9E-16
500,000	0.0277486	0.0001757	-3.9E-16
520,000	0.0279252	0.0001767	-4.0E-16
540,000	0.0281028	0.0001776	-4.0E-16
560,000	0.0282814	0.0001786	-4.1E-16
580,000	0.0284610	0.0001796	-4.1E-16
600,000	0.0286415	0.0001805	-4.2E-16
620,000	0.0288231	0.0001816	-4.2E-16

EXAMPLE CALCULATIONS FOR CONVERGENCE USING COORDINATES

Station coordinates - north 348,962.558 meters, east 829, 520.371 meters
West Zone

East coordinate of central meridian	850,000	meters
Subtract east coordinate of station	829,520	
X' =	20,480	

From table

			M
For North =		340,000	0.0263684
Actual	=	348,963	
Part		8,963	
Proportional part		8963/20000	
Percentage		45%	
Difference	=	0.0001694	
Difference times percentage			0.0000759
M Factor			**0.0264443**

From table

E Factor = **-3.6E-16**

CALC CONV

M times X'	=	542 sec.
E times X' cubed	=	-0 sec.
Sum		542 sec.
Convergence	=	**542 sec. or 9 min., 2 sec.**

Station coordinates - north 303,646.220 meters, east 860,950.555 meters
West Zone

East coordinate of central meridian	850,000	meters
Subtract east coordinate of station	860,951	
X' =	10,951	

From table

			M
For North =		300,000	0.0260321
Actual	=	303,646	
Part		3,646	
Proportional part		3646/20000	
Percentage		18%	
Difference	=	0.0001677	
Difference times percentage			0.0000306
M Factor			**0.0260627**

From table

E Factor = **-3.6E-16**

CALC CONV

M times X'	=	285 sec.
E times X' cubed	=	-0 sec.
Sum		285 sec.
Convergence	=	**285 sec. or 4 min., 45 sec.**

EXAMPLE CALCULATIONS FOR CONVERGENCE USING LATITUDE AND LONGITUDE

Station Coordinates in Missouri West Zone

	degrees	minutes	seconds
Latitude	39	18	38
Longitude	94	44	15
Central Meridian	94	30	0

(A) delta Longitude 0° 45' 45" = 855 sec
(B) sin Latitude 0.6335234214

Convergence = (A) * (B) 542 secs

degrees	minutes	seconds
0	-9	-2

Station Coordinates in Missouri Central Zone

	degrees	minutes	seconds
Latitude	39	18	38
Longitude	91	44	15
Central Meridian	92	30	0

(A) delta Longitude 0° 45' 45" = 2745 sec.
(B) sin Latitude p.6335234214

Convergence = (A) * (B) 1739 secs.

degrees	minutes	seconds
0	28	59

Station Coordinates in Missouri East Zone

	degrees	minutes	seconds
Latitude	39	18	38
Longitude	90	59	15
Central Meridian	90	30	0

(A) delta Longitude 0° 29' 15" = 1755 sec
(B) sin Latitude 0.6335234214

Convergence = (A) * (B) 1112 secs

degrees	minutes	seconds
0	-18	-32

CHAPTER 6
ARC-TO-CHORD CORRECTION

The land surveyor does not normally work with traverses of the length or high precision that requires using the arc-to-chord correction. Nevertheless, it is important for the land surveyor to understand when the correction should be made and how it is computed.

The surveyor measures on a curved surface but as we have seen, these measurements are projected on a plane surface for computation of the survey. The distance between two points on the curved ground surface is corrected to the plane by using the grid factor. A line connecting two points on the plane are straight lines but the line connecting the same points on the ground surface will be curved when projected on the plane. The surveyor actually measures the angle between the lines on the ground. They are the spherical angles and not the same angle that we use on the plane surface. The difference between the angle measured on the ground and the angle used on the plane is called the arc-to-chord or t-T correction. This correction is usually very small but may become significant for long lines near the edge of the zones. See figure 18.

The amount of curvature of the projected lines is dependent upon the distance from the central meridian and the direction of the line. In the Missouri Coordinate System, east-west lines project as straight lines and north-south lines project as curved lines. The north-south line at the central meridian is a straight line and increases in curvature as it moves east or west.

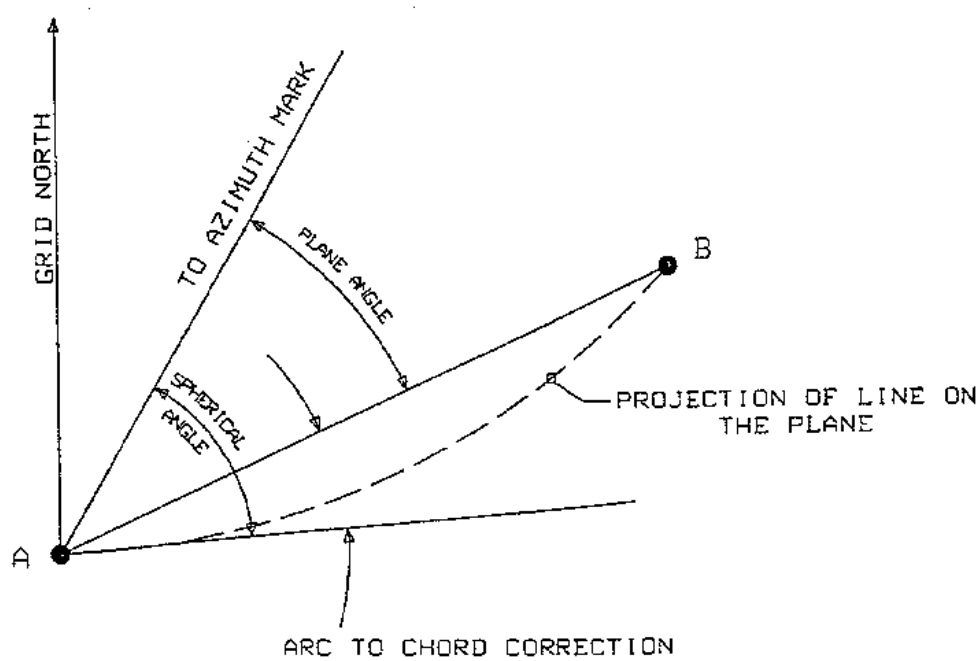


Figure 18. Arc to Chord Correction

The following example from Dr. Joe Senne, illustrates the procedure for using the arc-to-chord correction:

EXAMPLE TRAVERSE

Calculations for a traverse between two known points utilizing scale and elevation factors and the t-T (arc-to chord) correction.

This simulated traverse is located near the east edge of the Missouri central zone and runs from a south to northerly direction in order to maximize the t-T (arc-to-chord) correction. NAD83 SPC are used.

Because the lines are fairly long, a scale factor (SF) and elevation factor (EF) is computed for the midpoint of each line. The grid factor (GF) is equal to SF x EF.

An approximate but excellent expression is used to compute the t-T correction which is the difference between grid and projected geodetic azimuth of a line. The formula as shown is for the transverse mercator projection and automatically gives the correct sign.

$$(t\text{-}T) = .00254 \times X_o - X \times Y_2 - Y_1 \quad \text{(arc-seconds)}$$

Where X_o = Easting of the central meridian (km),
X = Easting of the midpoint of the line (km),
Y_1 = Northing of beginning of line (km),
Y_2 = Northing of end of line (km).

Since t-T usually amounts to only a few arc-seconds, the coordinates need only be measured to within about 500 meters, and can either be roughly computed or scaled from a map.

FIELD MEASUREMENTS

STA.	ANGLE RT. (° ' ")	AZIMUTH (° ' ")	NORTHING (meters)	EASTING (meters)	DIST. Meas. hori. (meters)	ELEV. (feet)
MK1						
		337 01 12 ***				
1	61 47 17		230335.536	586377.142**		1000
					7726.519	
2	286 05 06		236300.000	591200.000*		1050
					4650.277	
3	28 31 47		232500.000	593800.000*		920
					12283.604	
4	234 01 15		244700.000	592500.000*		887
					8926.324	
5	288 11 04		250792.215	599061.421**		950
		155 37 14***				
MK2						

* - Estimated values ** - Fixed values *** - (Fixed grid azimuth mark from North)

COMPUTE t-T CORRECTION AND CORRECT ANGLES

STA (° ' ")	ANGLE RT. (km)	U_2-Y_1 (km)	X_0-X BS (")	t-T (FS (")	t-T t-T (")	TOTAL ANGLE (")*	CORR.
MK1							
1	61 47 17			0.0	-1.4	-1.4	15.6
		6.00	-88.80				
2	286 05 06			-1.4	0.9	-0.5	05.5
		-3.80	-92.50				
3	28 31 47			0.9	-2.9	-2.0	45.0
		12.20	-93.15				
4	234 01 15			-2.9	-1.5	-4.4	10.6
		6.10	-95.75				
5	288 11 04			-1.5	0.0	-1.5	02.5
MK2							

* - Arc-seconds part of angle right

CALCULATE SF, EF, AND REDUCE MEASURED DISTANCE TO ELLIPSOID

Geoid height = 30.5m or -100.0 ft (below ellipsoid)

The scale factor (SF) can either be computed using the relationships between the ellipsoid and the mercator projection or by use of tables constructed specifically for each of the three zones in Missouri.

$$EF = \left(\frac{20909689}{29898689+H}\right) \quad 1{,}00000479$$

Where EF = elevation scale factor
H = elevation of surface at midpoint of line (feet)

Grid distance = SF x EF x measured distance.

LINE	SF	EF	GF	MEAS. DIST. (meters)	GRID DIST. (meters)
1-2	1.0000305	0.9999558	0.9999865	7726.519	7726.415
2-3	1.0000388	0.9999577	0.9999966	4650.277	4650.261
3-4	1.0000403	0.9999616	1.0000019	12283.604	12283.627
4-5	1.0000464	0.9999608	1.0000072	8926.324	8926.388

ADJUST TRAVERSE USING WEIGHTED LEAST SQUARES

Weighted Values:
Standard deviation of each angle = 3"
Constant part of EDM = 10 mm
Parts per million for EDM = 10

01-26-1992 22:08:59 HRS

INPUT DATA Data set No. 4
Traverse on East side of MO central zone with t-T grid angle correction

STA.	ANGLE RT. (d.ms)	AZIMUTH (d.ms)	NORTHING (meters)	EASTING	LINE	HOR DIST. (meters)
AZ REF						
		337.0112				
1	61.47156		230335.536	586377.142	1-2	7726.415
2	286.05055				2-3	4650.261
3	28.31450				3-4	12283.627
4	234.01106				4-5	8926.388
5	288.11025		250792.215	599061.421		
		155.3726				
AZ REF						

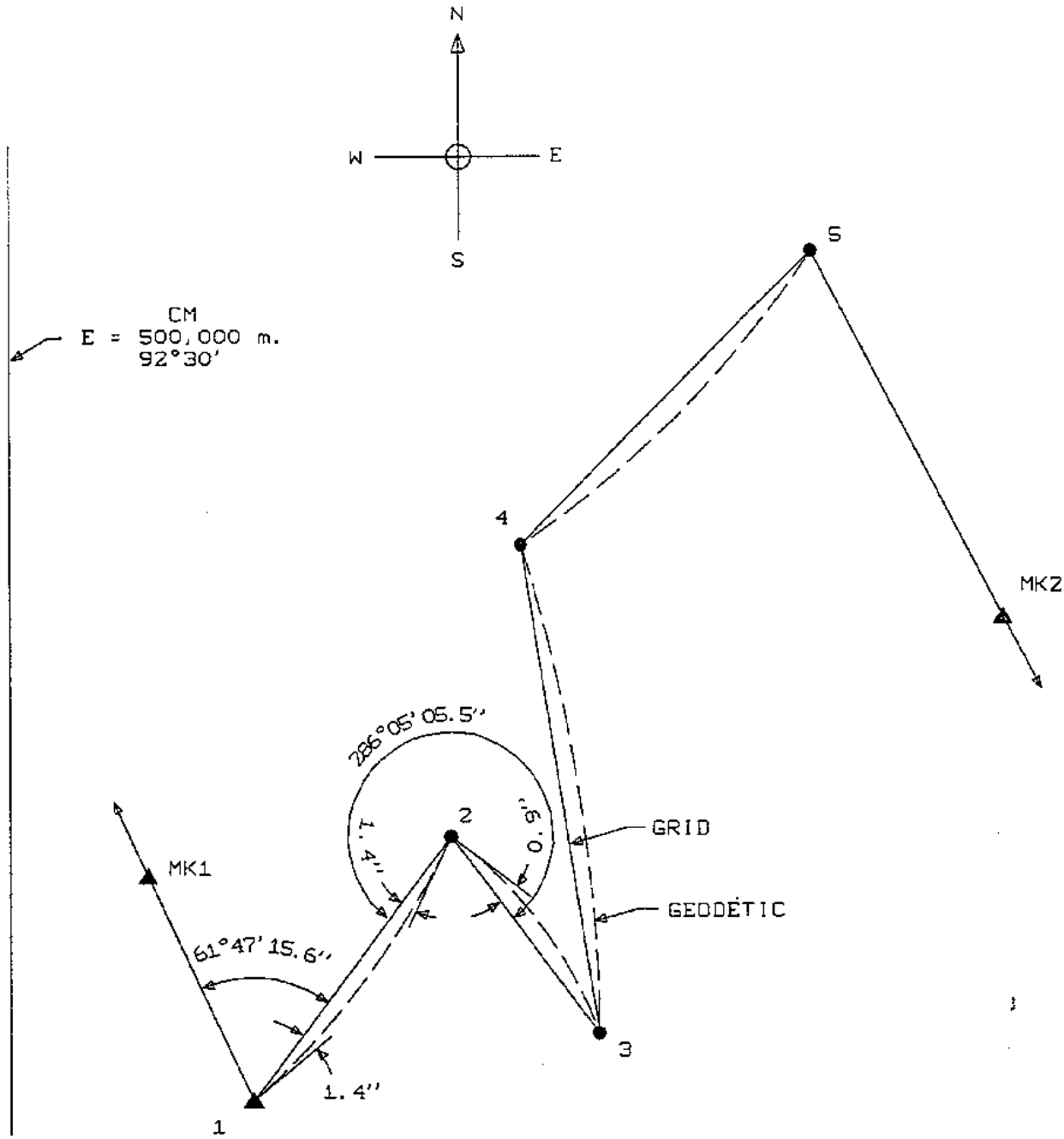
Azimuth closure in D.MS = 0.00052
Ratio of precision equals one part in 176554

INPUT DATA Data set No. 4
Traverse on East side of MO central zone with t-T grid angle correction

STA	NORTHING (meters)	EASTING (meters)	LINE	AZIMUTH (d m s)	DISTANCE (meters)
1	230335.536	586377.142	1-2	38 48 27.6	7726.406
2	236356.369	591219.344	2-3	144 53 32.4	4650.269
3	232552.110	593893.783	3-4	353 25 16.5	12283.582
4	244754.809	592486.469	4-5	47 26 25.8	8926.380

Sum = 33586.673

Ratio of precision before adjustment equals one part in 176554



TRAVERSE MISSOURI CENTRAL ZONE

Figure 19. Traverse Missouri Central Zone

Table 6 shows computed arc-to-chord corrections for various lengths of lines and various distances from the central meridian. Note that the correction is less than 1 second for lines less than 3 km and less than 130 km from the central meridian.

The land surveyor must determine when the accuracy, location, and size of the survey requires the use of the arc-to-chord correction.

TABLE 6 - SECOND TERM CORRECTION IN SECONDS

Distance from CM (in km)	Length of the Line in the North-South Direction (in km)				
	1	2	3	5	10
1	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.1	0.1
10	0.0	0.1	0.1	0.1	0.3
15	0.0	0.1	0.1	0.2	0.4
20	0.1	0.1	0.2	0.3	0.5
25	0.1	0.1	0.2	0.3	0.6
30	0.1	0.2	0.2	0.4	0.8
35	0.1	0.2	0.3	0.4	0.9
40	0.1	0.2	0.3	0.5	1.0
45	0.1	0.2	0.3	0.6	1.1
50	0.1	0.3	0.4	0.6	1.3
55	0.1	0.3	0.4	0.7	1.4
60	0.2	0.3	0.5	0.8	1.5
65	0.2	0.3	0.5	0.8	1.6
70	0.2	0.4	0.5	0.9	1.8
75	0.2	0.4	0.6	0.9	1.9
80	0.2	0.4	0.6	1.0	2.0
85	0.2	0.4	0.6	1.1	2.1
90	0.2	0.5	0.7	1.1	2.3
95	0.2	0.5	0.7	1.2	2.4
100	0.3	0.5	0.8	1.3	2.5
105	0.3	0.5	0.8	1.3	2.6
110	0.3	0.6	0.8	1.4	2.8
115	0.3	0.6	0.9	1.4	2.9
120	0.3	0.6	0.9	1.5	3.0
125	0.3	0.6	0.9	1.6	3.1
130	0.3	0.7	1.0	1.6	3.3

CHAPTER 7
MISSOURI COORDINATE SYSTEM 1983

The legal term for the state plane coordinate system that is being used is the "Missouri Coordinate System 1983." Many surveyors are beginning to use this coordinate system, so it is very important for all surveyors to understand the system and its use.

There are two reasons why surveyors are beginning to use state coordinates. There is more ground control available, because the Land Survey Program has been establishing and monumenting horizontal control in the metropolitan areas for many years. Just having more stations makes the system available to everyone. For example, control stations are available approximately every two miles in the many areas of the state. Cities and counties can now require that subdivision plats be tied into the state plane coordinate system. Another reason for the increased use of state coordinates is the availability of GPS. GPS not only makes the extension and densification of the geodetic control network possible but it allows the surveyor to establish state plane coordinates from the existing control with greater ease. It should be noted that the geodetic positions derived by use of the GPS are on the 1983 datum and consequently conversion to the Missouri Coordinate System places them on the Missouri Coordinate System 1983.

The primary and most common uses of the coordinate system is for mapping control. In order to meet the national and Missouri map accuracy standards, horizontal and vertical control must be in place on the ground. Without adequate existing control the photogrammetrist has to establish additional control for each mapping project. In addition to traditional mapping, the development of a Geographic Information System or computer map also relies on a Geographic Reference System such as the state plane coordinate system.

The newest use of the state plane coordinate system is as a reference for land survey corners. This facet of the state plane coordinate system is just coming into use and will have a very pronounced effect on the future of land surveying. The ability to accurately reference land corners by placing them on a coordinate system will be extremely important in years to come. In fact, there probably will be a time when the coordinate system will be the primary cadastre of the United States. But this will only come about when the surveyor and the public have confidence in their location.

Land surveyors need to be familiar with the state plane coordinate statute, Chapter 60.401 to 60.491. These sections define the state plane coordinate system and should be understood by all surveyors. The system is "marked on the ground by horizontal control stations established in conformity with the standards adopted by the Department of Natural Resources for first and second order quality work" (Chapter 60.451-3). This is an important concept. Stations must meet high quality standards in order to be used as the basis of the coordinate system. Stations that meet first or second order quality are defined as stations of the National Geodetic Reference System or stations of the Missouri Geographic Reference System.

All first and second order stations published by the U.S. Coast and Geodetic Survey or by the Department of Natural Resources meet the requirements defined in the statute. The Department of Natural Resources has adopted the Standards for this type of surveying. Ten CSR stations that do not meet these standards are not to be considered as a part of the Missouri Geographic Reference System.

There is a considerable difference between establishing Missouri Geographic Reference System stations and determining the state plane coordinates of a land corner. When the surveyor uses the coordinate system to define land boundaries, he must make a tie to a Missouri Geographic Reference System station; therefore, the coordinates are all based on the first or second order quality stations, but the actual coordinate value of the land owner does not have to meet first or second order quality. As stated previously, one of our primary goals at this time is to develop high public confidence and high positional accuracy of coordinates. The statute does not allow the surveyor to run more than one kilometer from a Geographic Reference System station to determine coordinates of a land corner. This criteria was based on the assumption that a transit traverse, meeting normal land survey quality would be accurate enough within the one kilometer range. By regulation, the land boundary corner must have a coordinate whose accuracy falls within the accuracy requirements of Minimum Standards for Property Boundary Surveys. It is entirely possible for the land surveyor to obtain the quality of state plane coordinates within a tenth of a foot for any traverse of one kilometer or less. If the surveyor has to run more than one kilometer, his traverse should be designed to obtain one tenth of a foot positional accuracy at the 67 percent confidence level. If the coordinate values are maintained at this accuracy level they will have a reasonable degree of resolution and will be accepted by the courts.

It is important to note the distinction between the quality of the coordinate on the property corner and the quality of the coordinate on the Missouri Geographic Reference System station. In order to obtain state plane coordinates on a property corner, it is not proper to begin at a property corner having a coordinate value. The coordinate must originate at a Missouri Geographic Reference System station.

There will be a greater and greater use of the coordinate system in the coming years. The advent of the GPS and other high precision surveying methods make it entirely appropriate to obtain the quality of a horizontal control station that we need. It is important for the surveyor to make sure that the coordinates he determines on land corners are in accordance with the Missouri Coordinate System of 1983.

Missouri Department of Natural Resources
Division of Geology and Land Survey
P.O. Box 250, Rolla, MO 65402-0250
(573) 368-2300

STATION NAME: PL-13,1988

STATE: Missouri COUNTY: Platte

TOWNSHIP: 52 North, RANGE: 34 West, SECTION: NE 20

USGS 7.5' QUAD: Ferrelview

NGS QUAD. OR QUIDQSN: 390943

DATE MONUMENTED: 1988

TECHNICAL DATA TABLE:

DATE OF OBSERVATIONS: 1988 ORDER: 1st
DATE OF ADJUSTMENT: 7/21/1989

GEODETIC DATA-NAD 1983:

LATITUDE: 39°18'38.35375"

LONGITUDE: 94°44'14.88803"

ELEVATION (METERS)*: 306.4
*NAVD 1988

ELEVATION DIFFERENCE (SURFACE MARK TO UNDERGROUND MARK)

	FEET	METERS
STATION:	2.98	0.908

STATE PLANE COORDINATE DATA:

MISSOURI

COORDINATES (METERS):

NORTH (Y): 348962.554

EAST (X): 829520.372

ZONE: MO-WEST

CONVERGENCE: -00°09'02"

GRID FACTOR: .9999031
(Note: 1 Meter = 3.28083333 Feet)

AZIMUTH MARK INFORMATION:

STATION	TO	STATION	ASTRONOMIC AZIMUTH	GRID AZIMUTH
PL-13, 1988	TO	PL-13A, 1988	271° 05' 45"	271° 14' 47"

Date of Report, 1996

STATION: PL-13,1988

DESCRIPTION:

STATION, AZIMUTH MARKS AND REFERENCE TIES:

The station is located in the NE 1/4 Section 20, T52N, R34W on the south R/W of NW 120th Street. It is located on a knoll 25 feet south of the centerline of road; 299 feet west of a private drive to a farmhouse and 160 feet east of a fence south.

The azimuth mark is located in the SE 1/4 SW 1/4 Section 17, T52N, R34W on the north R/W of NW 120th Street. It is 500 feet west of the intersection of Nevada Road and NW 120th Street; 26 feet north of the centerline of the road, 3.3 feet south of an E-W fence line and 29 feet east of a telephone pole.

STATION AND AZIMUTH MARK TO REACH:

To reach the station from the overpass of NW 120th Street and I-435, go east on NW 120th 1.12 miles to the station on right near E-W line fence.

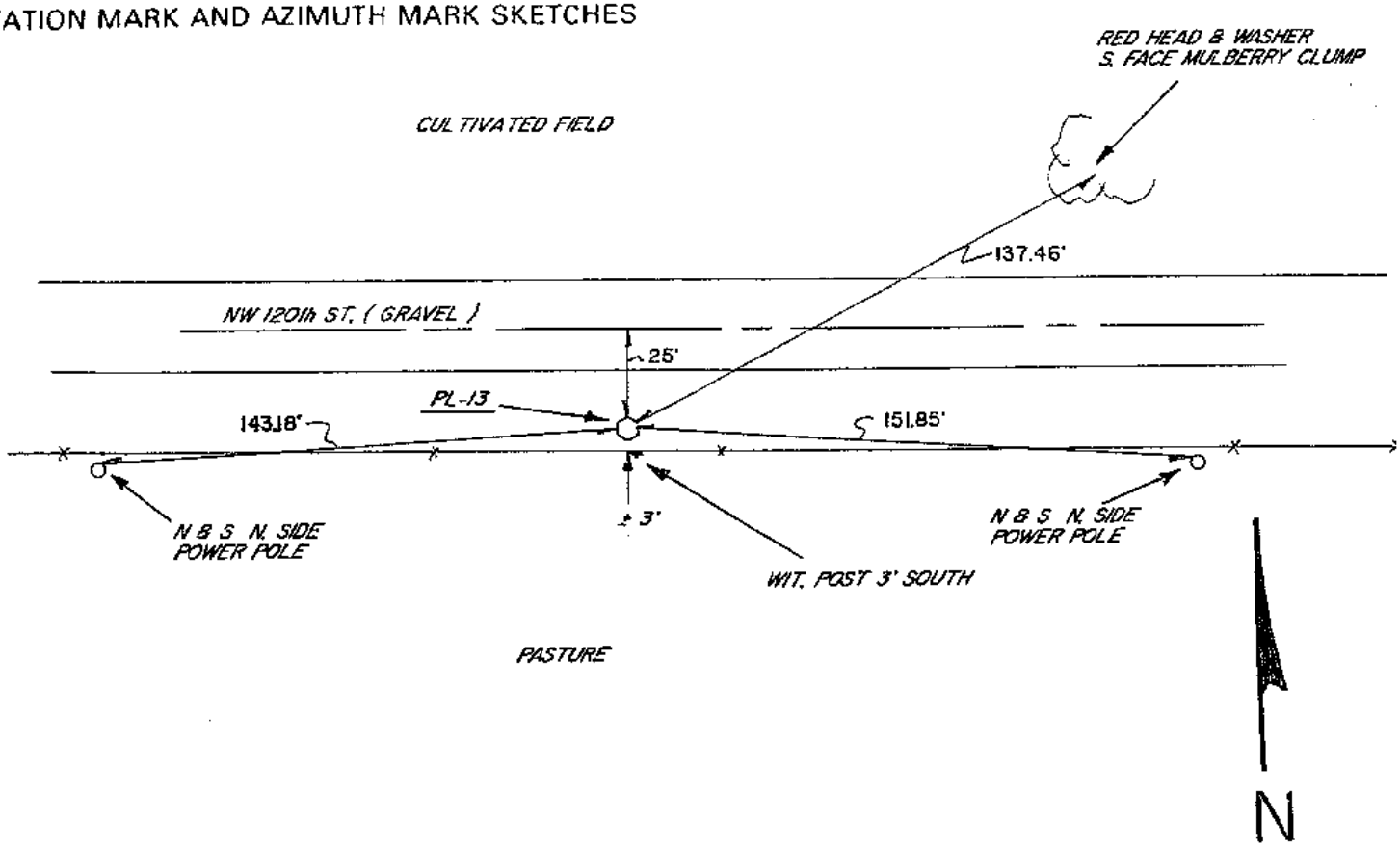
To reach the azimuth mark from the station go west on NW 120th Street (through intersection of Nevada and NW 120th Street) 0.37 miles to the azimuth mark on right.

Special Information:

Date of Report, 1996

STATION: PL-13 , 1988

STATION MARK AND AZIMUTH MARK SKETCHES



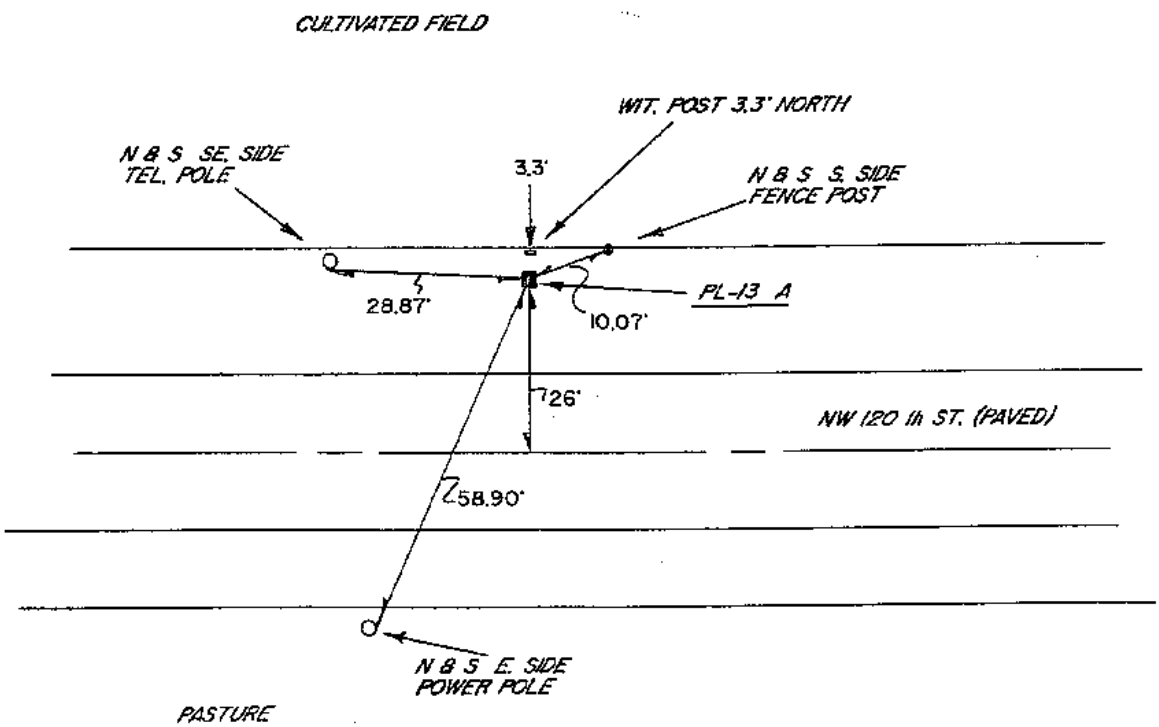


Figure 20. Station Diagram - PL-13

DATE OF SKETCH 1988

TRAVERSE
TO TIE SUBDIVISIONS CORNERS

Point	Grid Distance	Grid Bearing	Coordinate (meters)	
			North	East
JA-25A, No. 2				
		S81 20 50W		
JA-25			303646.224	860950.548
	237.168	S87 38 56W		
T-1			303636.50	860713.58
	1641.039	N60 08 48W		
T-2			304453.37	859290.30
	1274.539	N38 11 08W		
T-3			305455.18	858502.37
	400.954	N01 06 17E		
T-4			305856.06	858510.10
	142.895	N69 15 42E		
NW LOT 3A			305906.66	858643.74
	97.069	S03 24 06W		
SW LOT 3A			305809.76	858637.98
	3166.839	S46 54 25E		
JA-25			303646.224	860950.548

1 meter = 3.28083333'

All coordinates and all bearings shown in the traverse table are based on "Missouri Coordinate System of 1983, West Zone" using a grid factor of 0.999897.

Missouri Department of Natural Resources
Division of Geology and Land Survey
P.O. Box 250, Rolla, MO 65402-0250
(573) 368-2300

STATION NAME: JA-25, 1987

STATE: Missouri COUNTY: Jackson

TOWNSHIP: 47 North, RANGE: 31 West, SECTION: 8

USGS 7.5' QUAD: Lake Jacomo

NGS QUAD. OR QUIDQSN: 380941

DATE MONUMENTED: 1987

TECHNICAL DATA TABLE:

DATE OF OBSERVATIONS: 1988 ORDER: 1st
DATE OF ADJUSTMENT: 7/21/1989

GEODETIC DATA-NAD 1983:

LATITUDE: 38°54'09.40661"

LONGITUDE: 94°22'25.51534"

ELEVATION (METERS)*: 321.8

ELEVATION (FEET)*: 1055.7
*NAVD 1988

ELEVATION DIFFERENCE (SURFACE MARK TO UNDERGROUND MARK)
FEET METERS

STATION:

STATE PLANE COORDINATE DATA:

MISSOURI

COORDINATES (METERS):

NORTH (Y): 303646.224

EAST (X): 860950.548

ZONE: MO-WEST

CONVERGENCE: 0°04'45"

GRID FACTOR: .9998970
(Note: 1 Meter = 3.28083333 Feet)

AZIMUTH MARK INFORMATION:

STATION	TO	STATION	ASTRONOMIC AZIMUTH	GRID AZIMUTH
JA-25, 1987	TO	JA-25A, No. 2 1987-1989	81° 25' 35"	81° 20' 50"

Date of Report, 1996

STATION: JA-25, 1987

DESCRIPTION:

STATION, AZIMUTH MARKS AND REFERENCE TIES:

The station is located on the South R/W of U.S. Hwy 50, just south of the east end of the east bound on-ramp from Mo. Hwy 291. It is 52.16 feet east of a nail and shiner in the east face of a power pole; 26.91 feet WNW of a nail and shiner in the west face of a power pole; 24.41 feet NW of the NE corner of a block building (material storage building); 21.17 feet NE of the NW corner of the same block building.

The No. 2 azimuth mark is about 0.3 miles east on the north right-of-way of S.E. Blue Parkway near the southwest corner of Lee's Summit High School property. It is 26.8 feet west of the back of curb (exit for high school); 46.26 feet SSE of a nail and shiner in a power pole and 69.4 feet north of the centerline of S.E. Blue Parkway. The monument is a KC Metro disc set in a 12" poured in place concrete and stamped JA-25A No. 2 1987-1989.

STATION AND AZIMUTH MARK TO REACH:

To reach the station from the U.S. Hwy. 50 interchange with Mo. Hwy. 291 South (Jefferson St. north) in Lee's Summit, MO. go south on MO. 291 for 0.15 miles to the Realex Plant Entrance on the left (east). Turn left and go east for 0.05 miles to drive to main entrance to building leading north; turn left and follow drive northerly and easterly for 0.1 miles (across front and along north side of building) to a small block building on the left (north) side of the road and plant building. The monument is 20 +/- feet north of the block building on the south R/W of U.S. Hwy. 50 at the top of the backslope of a cut.

To reach the azimuth mark, JA-25A, No. 2, 1987-1989 from the center of U.S. 50 interchange with MO. Hwy. 291 (Jefferson St. North), go north to traffic light (S.E. Blue Parkway) turn right and go east 0.6 miles to the first entrance into Lee's Summit High School. Turn left and park near cyclone fence.

SPECIAL INFORMATION:

Access to station was granted from Realex Corporation (Mr. Bill Whitlow), 1001 S. 291 Hwy., Lee's Summit, Missouri (816)524-4160.

Date of Report, 1996

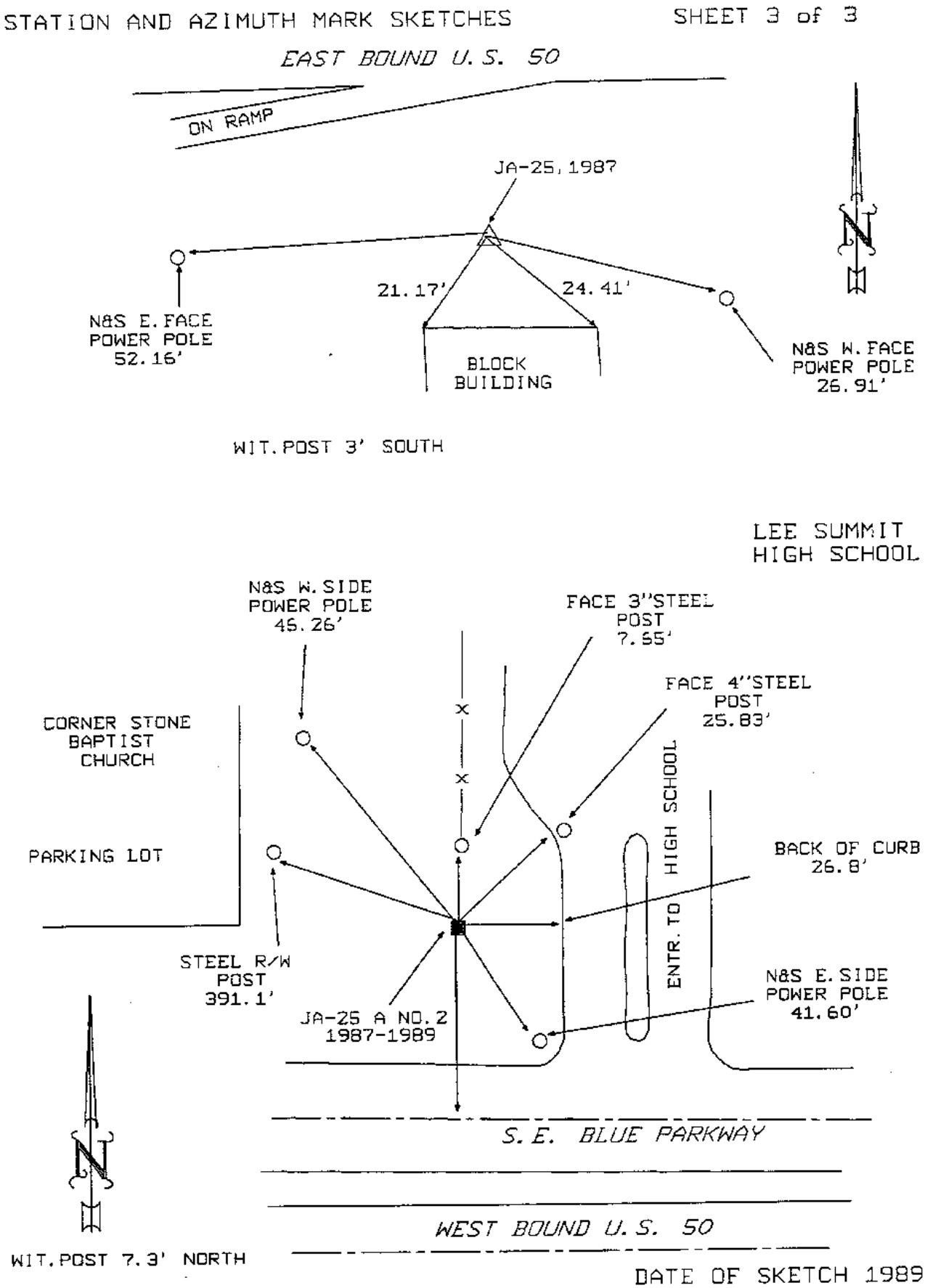


Figure 21. Station Diagram - JA-25

NOTES